Physics 505 **Fall 2005**

Homework Assignment #6 — Due Thursday, October 27

Textbook problems: Ch. 4: 4.2, 4.6 *a*) and *b*), 4.7 *a*) and *b*), 4.8

4.2 A point dipole with dipole moment $\vec{p}$ is located at the point $\vec{x}_0$. From the properties of the derivative of a Dirac delta function, show that for calculation of the potential Φ or the energy of a dipole in an external field, the dipole can be described by an effective charge density

$$\rho_{\rm eff}(\vec{x}\,) = -\vec{p}\cdot\nabla\delta(\vec{x}-\vec{x}_0)$$

4.6 A nucleus with quadrupole moment Q finds itself in a cylindrically symmetric electric field with a gradient $(\partial E_z/\partial z)_0$ along the z axis at the position of the nucleus.

a) Show that the energy of quadrupole interaction is

$$W = -\frac{e}{4}Q\left(\frac{\partial E_z}{\partial z}\right)_0$$

b) If it is known that $Q = 2\times10^{-28}\,\mathrm{m}^2$ and that W/h is 10 MHz, where h is Planck's constant, calculate $(\partial E_z/\partial z)_0$ in units of $e/4\pi\epsilon_0 a_0^3$, where $a_0 = 4\pi\epsilon_0\hbar^2/me^2 = 0.529\times10^{-10}\,\mathrm{m}$ is the Bohr radius in hydrogen.

4.7 A localized distribution of charge has a charge density

$$\rho(\vec{r}\,) = \frac{1}{64\pi}r^2e^{-r}\sin^2\theta$$

a) Make a multipole expansion of the potential due to this charge density and determine all the nonvanishing multipole moments. Write down the potential at large distances as a finite expansion in Legendre polynomials.

b) Determine the potential explicitly at any point in space, and show that near the origin, correct to r^2 inclusive,

$$\Phi(\vec{r}\,) \simeq \frac{1}{4\pi\epsilon_0}\left[\frac{1}{4}-\frac{r^2}{120}P_2(\cos\theta)\right]$$

4.8 A very long, right circular, cylindrical shell of dielectric constant ϵ/ϵ_0 and inner and outer radii a and b, respectively, is placed in a previously uniform electric field E_0 with its axis perpendicular to the field. The medium inside and outside the cylinder has a dielectric constant of unity.

a) Determine the potential and electric field in the three regions, neglecting end effects.

b) Sketch the lines of force for a typical case of $b \simeq 2a$.

c) Discuss the limiting forms of your solution appropriate for a solid dielectric cylinder in a uniform field, and a cylindrical cavity in a uniform dielectric.