Physics 505 **Fall 2005**

Homework Assignment #5 — Due Thursday, October 13

Textbook problems: Ch. 3: 3.14, 3.26, 3.27
Ch. 4: 4.1

3.14 A line charge of length $2d$ with a total charge Q has a linear charge density varying as $(d^2 - z^2)$, where z is the distance from the midpoint. A grounded, conducting, spherical shell of inner radius $b > d$ is centered at the midpoint of the line charge.

a) Find the potential everywhere inside the spherical shell as an expansion in Legendre polynomials.

b) Calculate the surface-charge density induced on the shell.

c) Discuss your answers to parts *a*) and *b*) in the limit that $d \ll b$.

3.26 Consider the Green function appropriate for Neumann boundary conditions for the volume V between the concentric spherical surfaces defined by $r = a$ and $r = b$, $a < b$. To be able to use (1.46) for the potential, impose the simple constraint (1.45). Use an expansion in spherical harmonics of the form

$$G(\vec{x}, \vec{x}\,') = \sum_{l=0}^{\infty} g_l(r, r') P_l(\cos\gamma)$$

where $g_l(r, r') = r_<^l / r_>^{l+1} + f_l(r, r')$.

a) Show that for $l > 0$, the radial Green function has the symmetric form

$$g_l(r, r') = \frac{r_<^l}{r_>^{l+1}} + \frac{1}{(b^{2l+1} - a^{2l+1})} \left[\frac{l+1}{l} (rr')^l + \frac{l}{l+1} \frac{(ab)^{2l+1}}{(rr')^{l+1}} + a^{2l+1} \left(\frac{r^l}{r'^{l+1}} + \frac{r'^l}{r^{l+1}} \right) \right]$$

b) Show that for $l = 0$

$$g_0(r, r') = \frac{1}{r_>} - \left(\frac{a^2}{a^2 + b^2} \right) \frac{1}{r'} + f(r)$$

where $f(r)$ is arbitrary. Show explicitly in (1.46) that answers for the potential $\Phi(\vec{x}\,)$ are independent of $f(r)$.

3.27 Apply the Neumann Green function of Problem 3.26 to the situation in which the normal electric field is $E_r = -E_0 \cos\theta$ at the outer surface ($r = b$) and is $E_r = 0$ on the inner surface ($r = a$).

a) Show that the electrostatic potential inside the volume V is

$$\Phi(\vec{x}) = E_0 \frac{r\cos\theta}{1-p^3}\left(1 + \frac{a^3}{2r^3}\right)$$

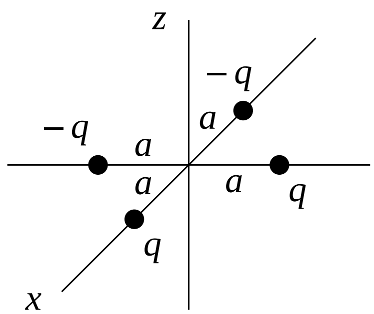
where $p = a/b$. Find the components of the electric field

$$E_r(r,\theta) = -E_0 \frac{\cos\theta}{1-p^3}\left(1 - \frac{a^3}{r^3}\right), \qquad E_\theta(r,\theta) = E_0 \frac{\sin\theta}{1-p^3}\left(1 + \frac{a^3}{2r^3}\right)$$

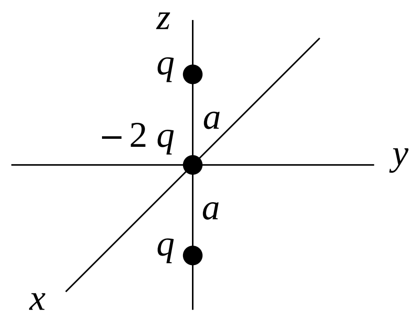
b) Calculate the Cartesian or cylindrical components of the field, E_z ad E_ρ, and make a sketch or computer plot of the lines of electric force for a typical case of $p = 0.5$.

4.1 Calculate the multipole moments q_{lm} of the charge distributions shown as parts *a*) and *b*). Try to obtain results for the nonvanishing moments valid for all l, but in each case find the first *two* sets of nonvanishing moments at the very least.

a)



b)



c) For the charge distribution of the second set *b*) write down the multipole expansion for the potential. Keeping only the lowest-order term in the expansion, plot the potential in the *x*-*y* plane as a function of distance from the origin for distances greater than a.

d) Calculate directly from Coulomb's law the exact potential for *b*) in the *x*-*y* plane. Plot it as a function of distance and compare with the result found in part *c*).

Divide out the asymptotic form in parts *c*) and *d*) to see the behavior at large distances more clearly.