Physics 505 **Fall 2005**

Homework Assignment #4 — Due Thursday, October 6

Textbook problems: Ch. 3: 3.4, 3.6, 3.9, 3.10

3.4 The surface of a hollow conducting sphere of inner radius a is divided into an *even number* of equal segments by a set of planes; their common line of intersection is the z axis and they are distributed uniformly in the angle ϕ. (The segments are like the skin on wedges of an apple, or the earth's surface between successive meridians of longitude.) The segments are kept at fixed potentials $\pm V$, alternately.

a) Set up a series representation for the potential inside the sphere for the general case of $2n$ segments, and carry the calculation of the coefficients in the series far enough to determine exactly which coefficients are different from zero. For the nonvanishing terms, exhibit the coefficients as an integral over $\cos\theta$.

b) For the special case of $n = 1$ (two hemispheres) determine explicitly the potential up to an including all terms with $l = 3$. By a coordinate transformation verify that this reduces to result (3.36) of Section 3.3.

3.6 Two point charges q and $-q$ are located on the z axis at $z = +a$ and $z = -a$, respectively.

a) Find the electrostatic potential as an expansion in spherical harmonics and powers of r for both $r > a$ and $r < a$.

b) Keeping the product $qa = p/2$ constant, take the limit of $a \to 0$ and find the potential for $r \neq 0$. This is by definition a dipole along the z axis and its potential.

c) Suppose now that the dipole of part *b*) is surrounded by a *grounded* spherical shell of radius b concentric with the origin. By linear superposition find the potential everywhere inside the shell.

3.9 A hollow right circular cylinder of radius b has its axis coincident with the z axis and its ends at $z = 0$ and $z = L$. The potential on the end faces is zero, while the potential on the cylindrical surface is given as $V(\phi, z)$. Using the appropriate separation of variables in cylindrical coordinates, find a series solution for the potential anywhere inside the cylinder.

3.10 For the cylinder in Problem 3.9 the cylindrical surface is made of two equal half-cylinders, one at potential V and the other at potential $-V$, so that

$$V(\phi, z) = \begin{cases} V & \text{for } -\pi/2 < \phi < \pi/2 \\ -V & \text{for } \pi/2 < \phi < 3\pi/2 \end{cases}$$

a) Find the potential inside the cylinder.

b) Assuming $L \gg b$, consider the potential at $z = L/2$ as a function of ρ and ϕ and compare it with two-dimensional Problem 2.13.