Physics 505 **Fall 2005**

Homework Assignment #3 — Due Thursday, September 29

Textbook problems: Ch. 2: 2.12, 2.13, 2.15
Ch. 3: 3.1

2.12 Starting with the series solution (2.71) for the two-dimensional potential problem with the potential specified on the surface of a cylinder of radius b, evaluate the coefficients formally, substitute them into the series, and sum it to obtain the potential *inside* the cylinder in the form of Poisson's integral:

$$\Phi(\rho,\phi) = \frac{1}{2\pi}\int_0^{2\pi} \Phi(b,\phi')\frac{b^2-\rho^2}{b^2+\rho^2-2b\rho\cos(\phi'-\phi)}\,d\phi'$$

What modification is necessary if the potential is desired in the region of space bounded by the cylinder and infinity?

2.13 *a*) Two halves of a long hollow conducting cylinder of inner radius b are separated by small lengthwise gaps on each side, and are kept at different potentials V_1 and V_2. Show that the potential inside is given by

$$\Phi(\rho,\phi) = \frac{V_1+V_2}{2} + \frac{V_1-V_2}{\pi}\tan^{-1}\left(\frac{2b\rho}{b^2-\rho^2}\cos\phi\right)$$

where ϕ is measured from a plane perpendicular to the plane through the gap.

b) Calculate the surface-charge density on each half of the cylinder.

2.15 *a*) Show that the Green function $G(x,y;x',y')$ appropriate for Dirichlet boundary conditions for a square two-dimensional region, $0 \le x \le 1$, $0 \le y \le 1$, has an expansion

$$G(x,y;x',y') = 2\sum_{n=1}^{\infty} g_n(y,y')\sin(n\pi x)\sin(n\pi x')$$

where $g_n(y,y')$ satisfies

$$\left(\frac{\partial^2}{\partial y'^2} - n^2\pi^2\right)g_n(y,y') = -4\pi\delta(y'-y) \quad \text{and} \quad g_n(y,0) = g_n(y,1) = 0$$

b) Taking for $g_n(y,y')$ appropriate linear combinations of $\sinh(n\pi y')$ and $\cosh(n\pi y')$ in the two regions, $y' < y$ and $y' > y$, in accord with the boundary conditions and the discontinuity in slope required by the source delta function, show that the explicit form of G is

$$G(x,y;x',y') = 8\sum_{n=1}^{\infty}\frac{1}{n\sinh(n\pi)}\sin(n\pi x)\sin(n\pi x')\sinh(n\pi y_<)\sinh[n\pi(1-y_>)]$$

where $y_<(y_>)$ is the smaller (larger) of y and y'.

3.1 Two concentric spheres have radii a, b ($b > a$) and each is divided into two hemispheres by the same horizontal plane. The upper hemisphere of the inner sphere and the lower hemisphere of the outer sphere are maintained at potential V. The other hemispheres are at zero potential.

Determine the potential in the region $a \le r \le b$ as a series in Legendre polynomials. Include terms at least up to $l = 4$. Check your solution against known results in the limiting cases $b \to \infty$, and $a \to 0$.