Physics 505 **Fall 2005**

Homework Assignment #2 — Due Thursday, September 22

Textbook problems: Ch. 2: 2.3, 2.4, 2.7, 2.10

2.3 A straight-line charge with constant linear charge density λ is located perpendicular to the x-y plane in the first quadrant at (x_0, y_0). The intersecting planes $x = 0$, $y \geq 0$ and $y = 0$, $x \geq 0$ are conducting boundary surfaces held at zero potential. Consider the potential, fields, and surface charges in the first quadrant.

a) The well-known potential for an isolated line charge at (x_0, y_0) is $\Phi(x, y) = (\lambda/4\pi\epsilon_0)\ln(R^2/r^2)$, where $r^2 = (x - x_0)^2 + (y - y_0)^2$ and R is a constant. Determine the expression for the potential of the line charge in the presence of the intersecting planes. Verify explicitly that the potential and the tangential electric field vanish on the boundary surfaces.

b) Determine the surface charge density σ on the plane $y = 0$, $x \geq 0$. Plot σ/λ versus x for $(x_0 = 2, y_0 = 1)$, $(x_0 = 1, y_0 = 1)$, and $(x_0 = 1, y_0 = 2)$.

c) Show that the total charge (per unit length in z) on the plane $y = 0$, $x \geq 0$ is

$$Q_x = -\frac{2}{\pi}\lambda \tan^{-1}\left(\frac{x_0}{y_0}\right)$$

What is the total charge on the plane $x = 0$?

d) Show that far from the origin [$\rho \gg \rho_0$, where $\rho = \sqrt{(x^2 + y^2)}$ and $\rho_0 = \sqrt{(x_0^2 + y_0^2)}$] the leading term in the potential is

$$\Phi \to \Phi_{\text{asym}} = \frac{4\lambda}{\pi\epsilon_0}\frac{(x_0 y_0)(xy)}{\rho^4}$$

Interpret.

2.4 A point charge is placed a distance $d > R$ from the center of an equally charged, isolated, conducting sphere of radius R.

a) Inside of what distance from the surface of the sphere is the point charge attracted rather than repelled by the charged sphere?

b) What is the limiting value of the force of attraction when the point charge is located a distance a $(= d - R)$ from the surface of the sphere, if $a \ll R$?

c) What are the results for parts *a*) and *b*) if the charge on the sphere is twice (half) as large as the point charge, but still the same sign?

2.7 Consider a potential problem in the half-space defined by $z \geq 0$, with Dirichlet boundary conditions on the plane $z = 0$ (and at infinity).

a) Write down the appropriate Green function $G(\vec{x}, \vec{x}\,')$.

b) If the potential on the plane $z = 0$ is specified to be $\Phi = V$ inside a circle of radius a centered at the origin, and $\Phi = 0$ outside that circle, find an integral expression for the potential at the point P specified in terms of cylindrical coordinates (ρ, ϕ, z).

c) Show that, along the axis of the circle ($\rho = 0$), the potential is given by

$$\Phi = V\left(1 - \frac{z}{\sqrt{a^2 + z^2}}\right)$$

d) Show that at large distances ($\rho^2 + z^2 \gg a^2$) the potential can be expanded in a power series in $(\rho^2 + z^2)^{-1}$, and that the leading terms are

$$\Phi = \frac{Va^2}{2}\frac{z}{(\rho^2 + z^2)^{3/2}}\left[1 - \frac{3a^2}{4(\rho^2 + z^2)} + \frac{5(3\rho^2 a^2 + a^4)}{8(\rho^2 + z^2)^2} + \cdots\right]$$

Verify that the results of parts *c*) and *d*) are consistent with each other in their common range of validity

2.10 A large parallel plate capacitor is made up of two plane conducting sheets with separation D, one of which has a small hemispherical boss of radius a on its inner surface ($D \gg a$). The conductor with the boss is kept at zero potential, and the other conductor is at a potential such that far from the boss the electric field between the plates is E_0.

a) Calculate the surface-charge densitites at an arbitrary point on the plane and on the boss, and sketch their behavior as a function of distance (or angle).

b) Show that the total charge on the boss has the magnitude $3\pi\epsilon_0 E_0 a^2$.

c) If, instead of the other conducting sheet at a different potential, a point charge q is placed directly above the hemispherical boss at a distance d from its center, show that the charge induced on the boss is

$$q' = -q\left[1 - \frac{d^2 - a^2}{d\sqrt{d^2 + a^2}}\right]$$