Physics 505 **Fall 2005**

Homework Assignment #1 — Due Thursday, September 15

Textbook problems: Ch. 1: 1.4, 1.5, 1.10, 1.14

1.4 Each of three charged spheres of radius a, one conducting, one having a uniform charge density within its volume, and one having a spherically symmetric charge density that varies radially as r^n ($n > -3$), has a total charge Q. Use Gauss' theorem to obtain the electric fields both inside and outside each sphere. Sketch the behavior of the fields as a function of radius for the first two spheres, and for the third with $n = -2, +2$.

1.5 The time-averagesd potential of a neutral hydrogen atom is given by

$$\Phi = \frac{q}{4\pi\epsilon_0}\frac{e^{-\alpha r}}{r}\left(1 + \frac{\alpha r}{2}\right)$$

where q is the magnitude of the electronic charge, and $\alpha^{-1} = a_0/2$, a_0 being the Bohr radius. Find the distribution of charge (both continuous and discrete) that will give this potential and interpret your result physically.

1.10 Prove the *mean value theorem*: For charge-free space the value of the electrostatic potential at any point is equal to the average of the potential over the surface of *any* sphere centered on that point.

1.14 Consider the electrostatic Green functions of Section 1.10 for Dirichlet and Neumann boundary conditions on the surface S bounding the volume V. Apply Green's theorem (1.35) with integration variable $\vec{y}$ and $\phi = G(\vec{x}, \vec{y}\,)$, $\psi = G(\vec{x}\,', \vec{y})$, with $\nabla_y^2 G(\vec{z}, \vec{y}\,) = -4\pi\delta(\vec{y} - \vec{z}\,)$. Find an expression for the difference $[G(\vec{x}, \vec{x}\,') - G(\vec{x}\,', \vec{x})]$ in terms of an integral over the boundary surface S.

a) For Dirichlet boundary conditions on the potential and the associated boundary condition on the Green function, show that $G_D(\vec{x}, \vec{x}\,')$ must be symmetric in $\vec{x}$ and $\vec{x}\,'$.

b) For Neumann boundary conditions, use the boundary condition (1.45) for $G_N(\vec{x}, \vec{x}\,')$ to show that $G_N(\vec{x}, \vec{x}\,')$ is not symmetric in general, but that $G_N(\vec{x}, \vec{x}\,') - F(\vec{x}\,)$ is symmetric in $\vec{x}$ and $\vec{x}\,'$, where

$$F(\vec{x}\,) = \frac{1}{S}\oint_S G_N(\vec{x}, \vec{y}\,)\, da_y$$

c) Show that the addition of $F(\vec{x}\,)$ to the Green function does not affect the potential $\Phi(\vec{x}\,)$. See problem 3.26 for an example of the Neumann Green function.