

Practice Final

The final will be a 180 minute open book, open notes exam. Do all four problems.

1. A point electric dipole with dipole moment \vec{p} is located at the center of a spherical hole of radius a inside a dielectric medium of infinite extent and dielectric constant ϵ/ϵ_0 .
 - a) Find the electric field everywhere.
 - b) What is the bound charge density on the surface of the dielectric?
2. A long straight wire carrying a current I is located in vacuum parallel to and a distance d away from a semi-infinite slab of permeability μ_r .
 - a) Find the magnetic induction everywhere.
 - b) What is the force per unit length on the wire (magnitude and direction)?
3. A rectangular parallel plate capacitor is formed of two flat rectangular conducting sheets of dimensions a and b separated by a distance d . A sinusoidal current $I(t) = I_0 e^{-i\omega t}$ is applied uniformly along the adjacent edges of length b . By orienting the plates so that they lie parallel to the x - y plane with the edge of length b parallel to the y axis, the electric field between the plates can be approximated by $\vec{E} = E_z(x)\hat{z}e^{-i\omega t}$.
 - a) Given the above electric field, show that the magnetic induction between the plates can only have a single component $\vec{B} = B_y(x)\hat{y}e^{-i\omega t}$.
 - b) Obtain exact expressions for $E_z(x)$ and $B_y(x)$ between the plates. Give your result in terms of Q , the maximum total charge on one plate.
 - c) The resonant frequency of this circuit occurs when the magnitude of the electric field is maximum on the side where the current is fed in and vanishing on the opposite side. What is the resonant frequency?
4. A plane polarized electromagnetic wave is incident with angle i on an interface between two dielectrics, with permittivities ϵ and ϵ' (take $\mu = \mu' = \mu_0$). This is the basic system investigated in section 7.3 of Jackson. The transmission and reflection coefficients are defined by

$$T = \left| \frac{E'_0}{E_0} \right|^2, \quad R = \left| \frac{E''_0}{E_0} \right|^2$$

- a) Show that in general $R + T \neq 1$. Consider both cases, \vec{E} perpendicular and parallel to the plane of incidence.
- b) Compute the time averaged Poynting vectors for the incident, reflected and refracted waves. Show that the power per unit area flowing in the direction normal to the interface is conserved. Again consider both polarization cases.