Physics 505 **Fall 2007**

Homework Assignment #10 — Due Thursday, November 29

Textbook problems: Ch. 6: 6.1, 6.4, 6.13, 6.18

6.1 In three dimensions the solution to the wave equation (6.32) for a point source in space and time (a light flash at $t' = 0, \vec{x}\,' = 0$) is a spherical shell disturbance of radius $R = ct$, namely the Green function $G^{(+)}$ (6.44). It may be initially surprising that in one or two dimensions, the disturbance possesses a "wake", even though the source is a "point" in space and time. The solutions for fewer dimensions than three can be found by superposition in the superfluous dimension(s), to eliminate dependence on such variable(s). For example, a flashing line source of uniform amplitude is equivalent to a point source in two dimensions.

a) Starting with the retarded solution to the three-dimensional wave equation (6.47), show that the source $f(\vec{x}\,', t) = \delta(x')\delta(y')\delta(t')$, equivalent to a $t = 0$ point source at the origin in two spatial dimensions, produces a two-dimensional wave

$$\Psi(x, y, t) = \frac{2c\Theta(ct - \rho)}{\sqrt{c^2t^2 - \rho^2}}$$

where $\rho^2 = x^2 + y^2$ and $\Theta(\xi)$ is the unit step function [$\Theta(\xi) = 0\ (1)$ if $\xi < (>)\ 0$.]

b) Show that a "sheet" source, equivalent to a point pulsed source at the origin in one space dimension, produces a one-dimensional wave proportional to

$$\psi(x, t) = 2\pi c\Theta(ct - |x|)$$

6.4 A uniformly magnetized and conducting sphere of radius R and total magnetic moment $m = 4\pi MR^3/3$ rotates about its magnetization axis with angular speed ω. In the steady state no current flows in the conductor. The motion is nonrelativistic; the sphere has no excess charge on it.

a) By considering Ohm's law in the moving conductor, show that the motion induces an electric field and a uniform volume charge density in the conductor, $\rho = -m\omega/\pi c^2R^3$.

b) Because the sphere is electrically neutral, there is no monopole electric field outside. Use symmetry arguments to show that the lowest possible electric multipolarity is quadrupole. Show that only a quadrupole field exists outside and that the quadrupole moment tensor has nonvanishing components, $Q_{33} = -4m\omega R^2/3c^2$, $Q_{11} = Q_{22} = -Q_{33}/2$.

c) By considering the radial electric fields inside and outside the sphere, show that the necessary surface-chrage density $\sigma(\theta)$ is

$$\sigma(\theta) = \frac{1}{4\pi R^2}\frac{4m\omega}{3c^2}\left[1 - \frac{5}{2}P_2(\cos\theta)\right]$$

d) The rotating sphere serves as a unipolar induction device if a stationary circuit is attached by a slip ring to the pole and a sliding contact to the equator. Show that the line integral of the electric field from the equator contact to the pole contact (by any path) is $\mathcal{E} = \mu_0 m\omega/4\pi R$.

6.13 A parallel plate capacitor is formed of two flat rectangular perfectly conducting sheets of dimensions a and b separated by a distance d small compared to a or b. Current is fed in and taken out uniformly along the adjacent edges of length b. With the input current and voltage defined at this end of the capacitor, calculate the input impedance or admittance using the field concepts of Section 6.9.

a) Calculate the electric and magnetic fields in the capacitor correct to second order in powers of the frequency, but neglecting fringing fields.

b) Show that the expansion of the reactance (6.140) in powers of the frequency to an appropriate order is the same as that obtained for a lumped circuit consisting of a capacitance $C = \epsilon_0 ab/d$ in series with an inductance $L = \mu_0 ad/3b$.

6.18 Consider the Dirac expression

$$\vec{A}(\vec{x}\,) = \frac{g}{4\pi}\int_L \frac{d\vec{l}\,' \times (\vec{x} - \vec{x}\,')}{|\vec{x} - \vec{x}\,'|^3}$$

for the vector potential of a magnetic monopole and its associated string L. Suppose for definiteness that the monopole is located at the origin and the string along the negative z axis.

a) Calculate $\vec{A}$ explicitly and show that in spherical coordinates it has components

$$A_r = 0, \qquad A_\theta = 0, \qquad A_\phi = \frac{g(1-\cos\theta)}{4\pi r\sin\theta} = \left(\frac{g}{4\pi r}\right)\tan\frac{\theta}{2}$$

b) Verify that $\vec{B} = \vec{\nabla}\times\vec{A}$ is the Coulomb-like field of a point charge, except perhaps at $\theta = \pi$.

c) With the $\vec{B}$ determined in part b, evaluate the total magnetic flux passing through the circular loop of radius $R\sin\theta$ shown in the figure. Consider $\theta < \pi/2$ and $\theta > \pi/2$ separately, but always calculate the upward flux.

d) From $\oint \vec{A}\cdot d\vec{l}$ around the loop, determine the total magnetic flux through the loop. Compare the result with that found in part c. Show that they are equal for $0 < \theta < \pi/2$, but have a *constant* difference for $\pi/2 < \theta < \pi$. Interpret this difference.