Physics 505 **Fall 2007**

Homework Assignment #8 — Due Thursday, November 8

Textbook problems: Ch. 5: 5.13, 5.14, 5.15, 5.16

5.13 A sphere of radius a carries a uniform surface-charge distribution σ. The sphere is rotated about a diameter with constant angular velocity ω. Find the vector potential and magnetic-flux density both inside and outside the sphere.

5.14 A long, hollow, right circular cylinder of inner (outer) radius a (b), and of relative permeability μ_r, is placed in a region of initially uniform magnetic-flux density $\vec{B}_0$ at right angles to the field. Find the flux density at all points in space, and sketch the logarithm of the ratio of the magnitudes of $\vec{B}$ on the cylinder axis to $\vec{B}_0$ as a function of $\log_{10}\mu_r$ for $a^2/b^2 = 0.5, 0.1$. Neglect end effects.

5.15 Consider two long, straight wires, parallel to the z axis, spaced a distance d apart and carrying currents I in opposite directions. Describe the magnetic field $\vec{H}$ in terms of a magnetic scalar potential Φ_M, with $\vec{H} = -\vec{\nabla}\Phi_M$.

a) If the wires are parallel to the z axis with positions, $x = \pm d/2$, $y = 0$, show that in the limit of small spacing, the potential is approximately that of a two-dimensional dipole

$$\Phi_M \approx -\frac{Id\sin\phi}{2\pi\rho} + \mathcal{O}(d^2/\rho^2)$$

where ρ and ϕ are the usual polar coordinates.

b) The closely spaced wires are now centered in a hollow right circular cylinder of steel, of inner (outer) radius a (b) and magnetic permeability $\mu = \mu_r\mu_0$. Determine the magnetic scalar potential in the three regions, $0 < \rho < a$, $a < \rho < b$, and $\rho > b$. Show that the field outside the steel cylinder is a two-dimensional dipole field, as in part a, but with a strength reduced by the factor

$$F = \frac{4\mu_r b^2}{(\mu_r+1)^2b^2 - (\mu_r-1)^2a^2}$$

Relate your result to Problem 5.14.

c) Assuming that $\mu_r \gg 1$, and $b = a + t$, where the thickness $t \ll b$, write down an approximate expression for F and determine its numerical value for $\mu_r = 200$ (typical of steel at 20 G), $b = 1.25$ cm, $t = 3$ mm. The shielding effect is relevant for reduction of stray fields in residential and commercial 60 Hz, 110 or 220 V wiring. [The figure illustrates the shielding effect for $a/b = 0.9$, $\mu_r = 100$.]

5.16 A circular loop of wire of radius a and negligible thickness carries a current I. The loop is centered in a spherical cavity of radius $b > a$ in a large block of soft iron. Assume that the relative permeability of the iron is effectively infinite and that of the medium in the cavity, unity.

a) In the approximation of $b \gg a$, show that the magnetic field at the center of the loop is augmented by a factor $(1 + a^3/2b^3)$ by the presence of the iron.

b) What is the radius of the "image" current loop (carrying the same current) that simulates the effect of the iron for $r < b$?