Physics 505 **Fall 2007**

Homework Assignment #5 — Due Thursday, October 11

Textbook problems: Ch. 3: 3.13, 3.17, 3.26, 3.27

3.13 Solve for the potential in Problem 3.1, using the appropriate Green function obtained in the text, and verify that the answer obtained in this way agrees with the direct solution from the differential equation.

3.17 The Dirichlet Green function for the unbounded space between the planes at $z = 0$ and $z = L$ allows discussion of a point charge or a distribution of charge between parallel conducting planes held at zero potential.

a) Using cylindrical coordinates show that one form of the Green function is

$$G(\vec{x}, \vec{x}\,') = \frac{4}{L}\sum_{n=1}^{\infty}\sum_{m=-\infty}^{\infty} e^{im(\phi-\phi')}\sin\left(\frac{n\pi z}{L}\right)\sin\left(\frac{n\pi z'}{L}\right) I_m\left(\frac{n\pi}{L}\rho_<\right) K_m\left(\frac{n\pi}{L}\rho_>\right)$$

b) Show that an alternative form of the Green function is

$$G(\vec{x}, \vec{x}\,') = 2\sum_{m=-\infty}^{\infty}\int_0^{\infty} dk\, e^{im(\phi-\phi')} J_m(k\rho) J_m(k\rho')\frac{\sinh(kz_<)\sinh[k(L-z_>)]}{\sinh(kL)}$$

3.26 Consider the Green function appropriate for Neumann boundary conditions for the volume V between the concentric spherical surfaces defined by $r = a$ and $r = b$, $a < b$. To be able to use (1.46) for the potential, impose the simple constraint (1.45). Use an expansion in spherical harmonics of the form

$$G(\vec{x}, \vec{x}\,') = \sum_{l=0}^{\infty} g_l(r, r') P_l(\cos\gamma)$$

where $g_l(r, r') = r_<^l / r_>^{l+1} + f_l(r, r')$.

a) Show that for $l > 0$, the radial Green function has the symmetric form

$$g_l(r, r') = \frac{r_<^l}{r_>^{l+1}} + \frac{1}{(b^{2l+1} - a^{2l+1})}\left[\frac{l+1}{l}(rr')^l + \frac{l}{l+1}\frac{(ab)^{2l+1}}{(rr')^{l+1}} + a^{2l+1}\left(\frac{r^l}{r'^{l+1}} + \frac{r'^l}{r^{l+1}}\right)\right]$$

b) Show that for $l = 0$

$$g_0(r, r') = \frac{1}{r_>} - \left(\frac{a^2}{a^2 + b^2}\right)\frac{1}{r'} + f(r)$$

where $f(r)$ is arbitrary. Show explicitly in (1.46) that answers for the potential $\Phi(\vec{x}\,)$ are independent of $f(r)$.

3.27 Apply the Neumann Green function of Problem 3.26 to the situation in which the normal electric field is $E_r = -E_0 \cos\theta$ at the outer surface $(r = b)$ and is $E_r = 0$ on the inner surface $(r = a)$.

a) Show that the electrostatic potential inside the volume V is

$$\Phi(\vec{x}\,) = E_0 \frac{r\cos\theta}{1 - p^3}\left(1 + \frac{a^3}{2r^3}\right)$$

where $p = a/b$. Find the components of the electric field

$$E_r(r, \theta) = -E_0 \frac{\cos\theta}{1 - p^3}\left(1 - \frac{a^3}{r^3}\right), \qquad E_\theta(r, \theta) = E_0 \frac{\sin\theta}{1 - p^3}\left(1 + \frac{a^3}{2r^3}\right)$$

b) Calculate the Cartesian or cylindrical components of the field, E_z ad E_ρ, and make a sketch or computer plot of the lines of electric force for a typical case of $p = 0.5$.