Physics 505 **Fall 2007**

Homework Assignment #4 — Due Thursday, October 4

Textbook problems: Ch. 3: 3.1, 3.2, 3.4, 3.7

3.1 Two concentric spheres have radii a, b ($b > a$) and each is divided into two hemispheres by the same horizontal plane. The upper hemisphere of the inner sphere and the lower hemisphere of the outer sphere are maintained at potential V. The other hemispheres are at zero potential.

Determine the potential in the region $a \le r \le b$ as a series in Legendre polynomials. Include terms at least up to $l = 4$. Check your solution against known results in the limiting cases $b \to \infty$, and $a \to 0$.

3.2 A spherical surface of radius R has charge uniformly distributed over its surface with a density $Q/4\pi R^2$, except for a spherical cap at the north pole, defined by the cone $\theta = \alpha$.

a) Show that the potential inside the spherical surface can be expressed as

$$\Phi = \frac{Q}{8\pi\epsilon_0} \sum_{l=0}^{\infty} \frac{1}{2l+1} [P_{l+1}(\cos\alpha) - P_{l-1}(\cos\alpha)] \frac{r^l}{R^{l+1}} P_l(\cos\theta)$$

where, for $l = 0$, $P_{l-1}(\cos\alpha) = -1$. What is the potential outside?

b) FInd the magnitude and the direction of the electric field at the origin.

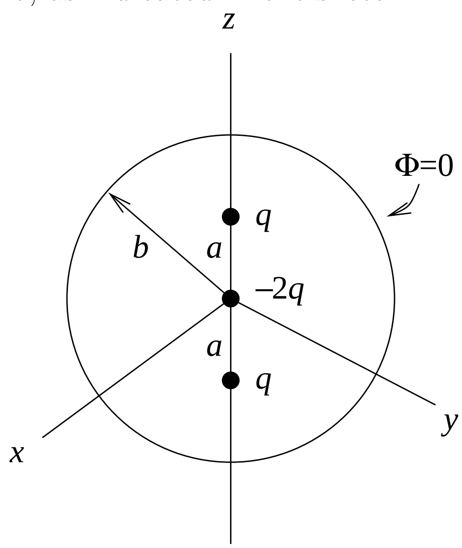
c) Discuss the limiting forms of the potential (part a) and electric field (part b) as the spherical cap becomes (1) very small, and (2) so large that the area with charge on it becomes a very small cap at the south pole.

3.4 The surface of a hollow conducting sphere of inner radius a is divided into an *even number* of equal segments by a set of planes; their common line of intersection is the z axis and they are distributed uniformly in the angle ϕ. (The segments are like the skin on wedges of an apple, or the earth's surface between successive meridians of longitude.) The segments are kept at fixed potentials $\pm V$, alternately.

a) Set up a series representation for the potential inside the sphere for the general case of $2n$ segments, and carry the calculation of the coefficients in the series far enough to determine exactly which coefficients are different from zero. For the nonvanishing terms, exhibit the coefficients as an integral over $\cos\theta$.

b) For the special case of $n = 1$ (two hemispheres) determine explicitly the potential up to and including all terms with $l = 3$. By a coordinate transformation verify that this reduces to result (3.36) of Section 3.3.

3.7 Three point charges $(q, -2q, q)$ are located in a straight line with separation a and with the middle charge $(-2q)$ at the origin of a grounded conducting spherical shell of radius b, as indicated in the sketch.



a) Write down the potential of the three charges in the absence of the grounded sphere. Find the limiting form of the potential as $a \to 0$, but the product $qa^2 = Q$ remains finite. Write this latter answer in spherical coordinates.

b) The presence of the grounded sphere of radius b alters the potential for $r < b$. The added potential can be viewed as caused by the surface-charge density induced on the inner surface at $r = b$ or by image charges located at $r > b$. Use linear superposition to satisfy the boundary conditions and find the potential everywhere inside the sphere for $r < a$ and $r > a$. Show that in the limit $a \to 0$,

$$\Phi(r, \theta, \phi) \to \frac{Q}{2\pi\epsilon_0 r^3}\left(1 - \frac{r^5}{b^5}\right) P_2(\cos\theta)$$