Physics 505 **Fall 2007**

Homework Assignment #3 — Due Thursday, September 27

Textbook problems: Ch. 2: 2.14, 2.15, 2.22, 2.23

2.14 A variant of the preceeding two-dimensional problem is a long hollow conducting cylinder of radius b that is divided into equal quarters, alternate segments being held at potential $+V$ and $-V$.

a) Solve by means of the series solution (2.71) and show that the potential inside the cylinder is

$$\Phi(\rho,\phi) = \frac{4V}{\pi}\sum_{n=0}^{\infty}\left(\frac{\rho}{b}\right)^{4n+2}\frac{\sin[(4n+2)\phi]}{2n+1}$$

b) Sum the series and show that

$$\Phi(\rho,\phi) = \frac{2V}{\pi}\tan^{-1}\left(\frac{2\rho^2 b^2\sin 2\phi}{b^4-\rho^4}\right)$$

c) Sketch the field lines and equipotentials.

2.15 *a*) Show that the Green function $G(x,y;x',y')$ appropriate for Dirichlet boundary conditions for a square two-dimensional region, $0\le x\le 1$, $0\le y\le 1$, has an expansion

$$G(x,y;x',y') = 2\sum_{n=1}^{\infty} g_n(y,y')\sin(n\pi x)\sin(n\pi x')$$

where $g_n(y,y')$ satisfies

$$\left(\frac{\partial^2}{\partial y'^2} - n^2\pi^2\right)g_n(y,y') = -4\pi\delta(y'-y) \quad \text{and} \quad g_n(y,0) = g_n(y,1) = 0$$

b) Taking for $g_n(y,y')$ appropriate linear combinations of $\sinh(n\pi y')$ and $\cosh(n\pi y')$ in the two regions, $y' < y$ and $y' > y$, in accord with the boundary conditions and the discontinuity in slope required by the source delta function, show that the explicit form of G is

$$G(x,y;x',y') = 8\sum_{n=1}^{\infty}\frac{1}{n\sinh(n\pi)}\sin(n\pi x)\sin(n\pi x')\sinh(n\pi y_<)\sinh[n\pi(1-y_>)]$$

where $y_<(y_>)$ is the smaller (larger) of y and y'.

2.22 *a*) For the example of oppositely charged conducting hemispherical shells separated by a tiny gap, as shown in Figure 2.8, show that the interior potential ($r < a$) on the z axis is

$$\Phi_{\text{in}}(z) = V\frac{a}{z}\left[1 - \frac{(a^2 - z^2)}{a\sqrt{a^2 + z^2}}\right]$$

Find the first few terms of the expansion in powers of z and show that they agree with (2.27) with the appropriate substitutions.

b) From the result of part a and (2.22), show that the radial electric field on the positive z axis is

$$E_r(z) = \frac{Va^2}{(z^2 + a^2)^{3/2}}\left(3 + \frac{a^2}{z^2}\right)$$

for $z > a$, and

$$E_r(z) = -\frac{V}{a}\left[\frac{3 + (a/z)^2}{(1 + (z/a)^2)^{3/2}} - \frac{a^2}{z^2}\right]$$

for $|z| < a$. Show that the second form is well behaved at the origin, with the value, $E_r(0) = -3V/2a$. Show that at $z = a$ (north pole inside) it has the value $-(\sqrt{2} - 1)V/a$. Show that the radial field at the north pole outside has the value $\sqrt{2}V/a$.

c) Make a sketch of the electric field lines both inside and outside the conducting hemispheres, with directions indicated. Make a *plot* of the radial electric field along the z axis from $z = -2a$ to $z = +2a$.

2.23 A hollow cube has conducting walls defined by six planes $x = 0$, $y = 0$, $z = 0$, and $x = a$, $y = a$, $z = a$. The walls $z = 0$ and $z = a$ are held at a constant potential V. The other four sides are at zero potential.

a) Find the potential $\Phi(x, y, z)$ at any point inside the cube.

b) Evaluate the potential at the center of the cube numerically, accurate to three significant figures. How many terms in the series is it necessary to keep in order to attain this accuracy? Compare your numerical result with the average value of the potential on the walls. See Problem 2.28.

c) Find the surface-charge density on the surface $z = a$.