Physics 505 **Fall 2007**

Homework Assignment #2 — Due Thursday, September 20

Textbook problems: Ch. 1: 2.2, 2.8, 2.10, 2.11

2.2 Using the method of images, discuss the problem of a point charge q *inside* a hollow, grounded, conducting sphere of inner radius a. Find

a) the potential inside the sphere;

b) the induced surface-charge density;

c) the magnitude and direction of the force acting on q.

d) Is there any change in the solution if the sphere is kept at a fixed potential V? If the sphere has a total charge Q on its inner and outer surfaces?

2.8 A two-dimensional potential problem is defined by two straight parallel line charges separated by a distance R with equal and opposite linear charge densities λ and $-\lambda$.

a) Show by direct construction that the surface of constant potential V is a circular cylinder (circle in the transverse dimensions) and find the coordinates of the axis of the cylinder and its radius in terms of R, λ, and V.

b) Use the results of part a to show that the capacitance per unit length C of two right-circular cylindrical conductors, with radii a and b, separated by a distance $d > a + b$, is

$$C = \frac{2\pi\epsilon_0}{\cosh^{-1}\left(\dfrac{d^2 - a^2 - b^2}{2ab}\right)}$$

c) Verify that the result for C agrees with the answer in Problem 1.7 in the appropriate limit and determine the next nonvanishing order correction in powers of a/d and b/d.

d) Repeat the calculation of the capacitance per unit length for two cylinders inside each other ($d < |b - a|$). Check the result for concentric cylinders ($d = 0$).

2.10 A large parallel plate capacitor is made up of two plane conducting sheets with separation D, one of which has a small hemispherical boss of radius a on its inner surface ($D \gg a$). The conductor with the boss is kept at zero potential, and the other conductor is at a potential such that far from the boss the electric field between the plates is E_0.

a) Calculate the surface-charge densitites at an arbitrary point on the plane and on the boss, and sketch their behavior as a function of distance (or angle).

b) Show that the total charge on the boss has the magnitude $3\pi\epsilon_0 E_0 a^2$.

c) If, instead of the other conducting sheet at a different potential, a point charge q is placed directly above the hemispherical boss at a distance d from its center,

show that the charge induced on the boss is

$$q' = -q\left[1 - \frac{d^2 - a^2}{d\sqrt{d^2 + a^2}}\right]$$

2.11 A line charge with linear charge density τ is placed parallel to, and a distance R away from, the axis of a conducting cylinder of radius b held at fixed voltage such that the potential vanishes at infinity. Find

a) the magnitude and position of the image charge(s);

b) the potential at any point (expressed in polar coordinates with the origin at the axis of the cylinder and the direction from the origin to the line charge as the x axis), including the asymptotic form far from the cylinder;

c) the induced surface-charge density, and plot it as a function of angle for $R/b = 2$, 4 in units of $\tau/2\pi b$;

d) the force per unit length on the line charge.