Textbook problems: Ch. 1: 1.5, 1.7, 1.11, 1.12

1.5 The time-averaged potential of a neutral hydrogen atom is given by

\[ \Phi = \frac{q}{4\pi \epsilon_0} \frac{e^{-\alpha r}}{r} \left(1 + \frac{\alpha r}{2}\right) \]

where \( q \) is the magnitude of the electronic charge, and \( \alpha^{-1} = a_0/2 \), \( a_0 \) being the Bohr radius. Find the distribution of charge (both continuous and discrete) that will give this potential and interpret your result physically.

1.7 Two long, cylindrical conductors of radii \( a_1 \) and \( a_2 \) are parallel and separated by a distance \( d \), which is large compared with either radius. Show that the capacitance per unit length is given approximately by

\[ C = \pi \epsilon_0 \left( \ln \frac{d}{a} \right)^{-1} \]

where \( a \) is the geometrical mean of the two radii.

Approximately what gauge wire (state diameter in millimeters) would be necessary to make a two-wire transmission line with a capacitance of \( 1.2 \times 10^{-11} \) F/m if the separation of the wires was 0.5 cm? 1.5 cm? 5.0 cm?

1.11 Use Gauss’s theorem to prove that at the surface of a curved charged conductor, the normal derivative of the electric field is given by

\[ \frac{1}{E} \frac{\partial E}{\partial n} = - \left( \frac{1}{R_1} + \frac{1}{R_2} \right) \]

where \( R_1 \) and \( R_2 \) are the principal radii of curvature of the surface.

1.12 Prove Green’s reciprocation theorem: If \( \Phi \) is the potential due to a volume-charge density \( \rho \) within a volume \( V \) and a surface-charge density \( \sigma \) on the conducting surface \( S \) bounding the volume \( V \), while \( \Phi' \) is the potential due to another charge distribution \( \rho' \) and \( \sigma' \), then

\[ \int_V \rho \Phi' \, d^3x + \int_S \sigma \Phi' \, da = \int_V \rho' \Phi \, d^3x + \int_S \sigma' \Phi \, da \]