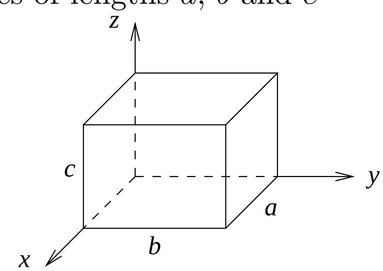
Physics 505 **Fall 2007**

Practice Midterm

The midterm will be a two hour open book, open notes exam. Do all three problems.

1. A rectangular box has sides of lengths a, b and c



a) For the Dirichlet problem in the interior of the box, the Green's function may be expanded as

$$G(x,y,z;x',y',z') = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} g_{mn}(z,z') \sin\frac{m\pi x}{a} \sin\frac{m\pi x'}{a} \sin\frac{n\pi y}{b} \sin\frac{n\pi y'}{b}$$

Write down the appropriate differential equation that $g_{mn}(z,z')$ must satisfy.

b) Solve the Green's function equation for $g_{mn}(z,z')$ subject to Dirichlet boundary conditions and write down the result for $G(x,y,z;x',y',z')$.

c) Consider the boundary value problem where the potential on top of the box is $\Phi(x,y,c) = V(x,y)$ while the potential on the other five sides vanish. Using the Greens' function obtained above, show that the potential may be written as

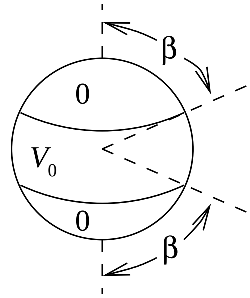
$$\Phi(x,y,z) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \sinh \gamma_{mn} z$$

where $\gamma_{mn} = \pi\sqrt{(m/a)^2 + (n/b)^2}$ and

$$A_{mn} = \frac{4}{ab \sinh \gamma_{mn} c} \int_0^a dx \int_0^b dy\, V(x,y) \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}$$

Over ⟶

2. The potential on the surface of a sphere of radius a is specified by



$$V(\theta,\phi) = \begin{cases} 0, & 0 \le \theta < \beta \\ V_0, & \beta \le \theta \le \pi - \beta \\ 0, & \pi - \beta < \theta \le \pi \end{cases}$$

There are no other charges in this problem.

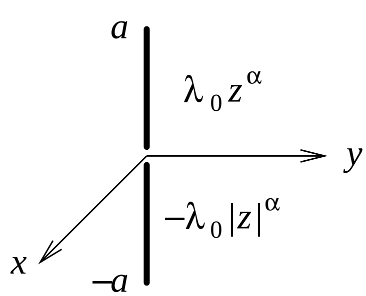
a) Show that the potential outside the sphere may be expressed as

$$\Phi(r,\theta,\phi) = \sum_{l=0,2,4,6,\ldots} V_0[P_{l+1}(\cos\beta) - P_{l-1}(\cos\beta)] \left(\frac{a}{r}\right)^{l+1} P_l(\cos\theta)$$

where we take $P_{-1}(x) = 0$. Note that Legendre polynomials satisfy the relation $(2l+1)P_l(x) = P'_{l+1}(x) - P'_{l-1}(x)$.

b) For fixed V_0, what angle β maximizes the quadrupole moment?

3. A line charge on the z axis extends from $z = -a$ to $z = +a$ and has linear charge density varying as



$$\lambda(z) = \begin{cases} \lambda_0 z^\alpha, & 0 < z \le a \\ -\lambda_0 |z|^\alpha, & -a \le z < 0 \end{cases}$$

where α is a positive constant. The total charge on the $0 < z \le a$ segment is Q (and the charge on the $-a \le z < 0$ segment is $-Q$).

a) Calculate all of the multipole moments of the charge distribution. Make sure to indicate which moments are non-vanishing.

b) Write down the multipole expansion for the potential in explicit form up to the first two non-vanishing terms.

c) What is the dipole moment $\vec{p}$ in terms of Q, a and α?