Physics 505 **Fall 2007**

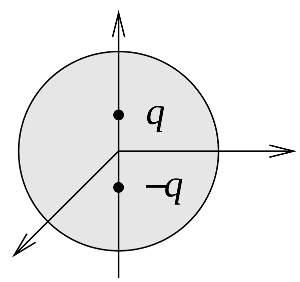
Practice Final

The final will be a three hour open book, open notes exam. Do all four problems.

1. Two point charges q and $-q$ are located on the z axis at $z = +d/2$ and $z = -d/2$, respectively.

   a) If the charges are isolated in space, show that the potential admits a Legendre expansion
   $$\Phi(r, \theta) = \frac{2q}{4\pi\epsilon_0} \sum_{l \text{ odd}} \frac{r_<^l}{r_>^{l+1}} P_l(\cos\theta)$$
   where $r_< = \min(r, d/2)$ and $r_> = \max(r, d/2)$.

   b) Now consider the charges to be contained inside a linear dielectric sphere of permittivity ϵ and radius a (where $a > d/2$).

   Find the electric potential everywhere as an expansion in Legendre polynomials.

2. A point magnetic dipole $\vec{m}$ is located in vacuum pointing away from and a distance d away from a semi-infinite slab of material with relative permeability μ_r.

   a) Find the magnetic induction everywhere.

   b) What is the force on the dipole (magnitude and direction)?

3. An infinitely long solenoid of radius a has N tightly wound turns per unit length. For a constant current I, elementary considerations tells us that the magnetic induction is uniform inside the solenoid. In cylindrical coordinates, $\vec{B} = \mu_0 N I \hat{z} \Theta(a - \rho)$ where $\Theta(\xi) = 1$ if $\xi > 0$ (and 0 otherwise) is the unit step function.

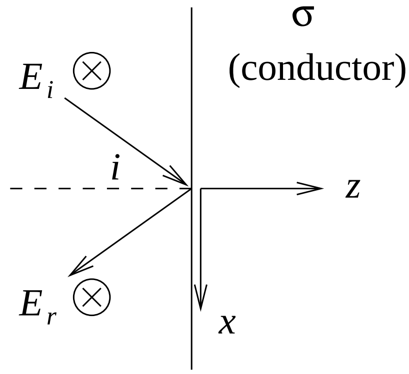
   This problem, however, involves a sinusoidal current $I(t) = I_0 e^{-i\omega t}$. In the following, consider only the inside of the solenoid and assume all fields vanish outside.

   a) By symmetry considerations, the time-dependent magnetic induction only has a non-vanishing z component, $B_z(\rho)e^{-i\omega t}$. Show that the electric field only has a component along the $\hat{\phi}$ direction. Consider the inside of the solenoid only.

   b) Find the exact solution for $B_z(\rho)$ inside the solenoid. Give your result in terms of the maximum current I flowing through the wires of the solenoid.

Over ⟶

4. A plane polarized electromagnetic wave of frequency ω in free space is incident with angle i on the flat surface of an excellent conductor ($\mu = \mu_0$, $\epsilon = \epsilon_0$ and $\sigma \gg \omega\epsilon_0$) which fills the region $z > 0$.



Consider *only* linear polarization perpendicular to the plane of incidence.

a) If the incident wave is given by $\vec{E} = \vec{E}_i e^{i(\vec{k}\cdot\vec{x}-\omega t)}$, show that (in the limit $\sigma \gg \omega\epsilon_0$) the magnitude of the electric field inside the conductor is

$$E_c = E_i \gamma \cos i \, e^{-z/\delta} e^{i(kx \sin i + z/\delta - \omega t)}$$

where

$$\delta = \sqrt{\frac{2}{\omega\mu_0\sigma}} \qquad \text{and} \qquad \gamma = (1-i)\sqrt{\frac{2\epsilon_0\omega}{\sigma}}$$

The z direction is perpendicular to the flat surface of the conductor, while the x direction is parallel to it.

b) Show that the time averaged power per unit area flowing into the conductor is given by $S^{\perp} = \epsilon_0 |E_i|^2 \omega\delta \cos^2 i$.