

MODELING CHURN AND USAGE BEHAVIOR IN CONTRACTUAL SETTINGS

Eva Ascarza
Bruce G. S. Hardie[†]

March 2009

[†]Eva Ascarza is a doctoral candidate in Marketing at London Business School (email: eascarza@london.edu; web: www.evaascarza.com). Bruce G. S. Hardie is Professor of Marketing, London Business School (email: bhardie@london.edu; web: www.brucehardie.com). The authors thank Naufel Vilcassim for his helpful comments on an earlier version of this paper, and acknowledge the support of the London Business School Centre for Marketing.

Abstract

Modeling Churn and Usage Behavior in Contractual Settings

The ability to retain existing customers is a major concern for many businesses. However retention is not the only dimension of interest; the revenue stream associated with each customer is another key factor influencing customer profitability.

In most contractual situations the exact revenue that will be generated per customer is uncertain at the beginning of the contract period; customer revenue is determined by how much of the service each individual consumes. While a number of researchers have explored the problem of modeling retention in a contractual setting, the literature has been surprisingly silent on how to forecast customers' usage (and therefore future revenue) in contractual situations.

We propose a dynamic latent trait model in which usage and renewal behavior are modeled simultaneously by assuming that both behaviors are driven by the same (individual-level) underlying process that evolves over time. We capture the dynamics in the underlying latent variable (which we label "commitment") using a hidden Markov model, and then incorporate unobserved heterogeneity in the usage process.

The model parameters are estimated using hierarchical Bayesian methods. We validate the model using data from a so-called Friends scheme run by a performing arts organization. First we show how the proposed model outperforms benchmark models on both the usage and retention dimensions. In contrast to most churn models, this dynamic model is able to identify changes in behavior before the contract is close to expiring, thus providing early predictions of churn. Moreover, the model provides additional insights into the behavior of the customer base that are of interest to managers.

Keywords: Contractual settings, hidden Markov model, RFM, Bayesian estimation, latent variable models.

“Retention consists of two components — will the customer stay with the company and how much will the customer spend. The “staying” aspect has received much more attention than the spending aspect, but they both need to be modeled.” (Blattberg et al. 2008, p. 690)

1 Introduction

The ability to retain existing customers is a major concern for many businesses, especially in mature industries where customer acquisition is very costly and the competitive environment is rather severe (Blattberg et al. 2001, Rust et al. 2001). One way to increase retention is to identify which customers are most likely to churn, and then undertake targeted marketing campaigns designed to encourage them to stay. Hence, early detection of potential churners can reduce defection, thus increasing business profitability (Bolton et al. 2004, Reinartz et al. 2005). Moreover, in many contractual business settings, retention is not the only dimension of interest in the customer relationship; there are other behaviors that drive customer profitability. This is the case in settings where we observe customer *usage* while “under contract.” Examples include mobile phone services (where we observe the number of calls made), gym memberships (where we observe visits to the gym, number of extra classes attended, etc.), online magazine subscriptions (where log-ins are observed) and “friends” schemes for arts organization (where we observe the number of exhibitions or performances attended). In such business settings, predicting future usage is an important input into any analysis of customer profitability.¹

The objective of this paper is to develop a model that can forecast both behaviors; we are interested in predicting renewal and usage behavior in those settings where usage is not known in advance but is of managerial interest, either because it directly affects revenue, or because it affects service quality, which in turn affects customer retention (Nam et al. 2007). Such a model must accommodate several aspects that are common across

¹We acknowledge that in some situations customer revenue is known in advance and thus independent of usage. This is the case of flat-fee or “all inclusive” contracts, where customers’ revenue is fixed. Even in such cases, forecasting customer usage may be of interest to the firm because it affects service quality. For example, consider a broadband provider offering flat-fee contracts: if the company does not manage to predict usage accurately, it could face capacity problems when many customers connect at the same time, thus reducing the quality of their connections.

contractual settings. First, one of the variables of interest is binary (renewal is always a “yes” or “no” decision) whereas the other is not (e.g., number of transactions is a count variable). Second, the renewal process is absorbing. That is, once a customer churns she cannot use the service in any future period (unless she takes out a new contract). Third, the model should allow the renewal and usage processes to occur on different time scales. This is a very common pattern in contractual businesses. For example, let us consider a gym offering monthly memberships and summarizing attendance on a weekly basis, or a mobile phone operator with monthly contracts in which weekly consumption is recorded. In these two situations, while usage is observed on a weekly basis, renewal only happens at the end of each month. Ignoring intra-month usage would be a waste of useful information that can enrich the model predictions. And finally, the model must require only information that can be extracted easily from the firm’s database. The last point is not unique to contractual settings but is a realistic requirement for the model to be used in practice.

At first glance, a discrete/continuous model of consumer demand (e.g., Chintagunta 1993, Hanemann 1984, Krishnamurthi and Raj 1988) appears to be an obvious starting point. This type of model was proposed in the marketing and economics literature to model binary/continuous decisions, such as “whether to buy” and if so “how much to buy.” More recently these models have been extended to accommodate dropout (e.g., Narayanan et al. 2007, Ascarza et al. 2009). However, these models do not accommodate the two different time scales and more importantly, since they are based on a utility maximizing framework with stable preferences over time, they are more appropriate to *explain* rather than *forecast* customers decisions.

The problem of modeling retention has received much attention from both academics and practitioners. Researchers working in the areas of marketing, applied statistics, and data mining have developed a number of models that attempt to either explain or predict churn (e.g., Bhattacharya 1998, Kim and Yoon 2004, Larivière and Van den Poel 2005, Lemon et al. 2002, Lu 2002, Mozer et al. 2000, Parr Rud 2001, Schweidel et al. 2008). One stream of work has sought to model churn as a function of data readily available in the firm’s databases, such as marketing activities, demographics, and past customer

behavior. (See Blattberg et al. (2008) for a review of these various methods.) What we observe is that past usage behavior is an important predictor variable (Figure 1a). Another stream of work has explored the link between customers' attitudes towards the service and subsequent churn behavior (Figure 1b). Bolton (1998) shows that satisfaction levels explain a substantial portion of the variance in contract durations. Athanassopoulos (2000) and Verhoef (2003) find that affective commitment is positively related to contract duration.

There is limited research on the modeling of service usage in contractual settings. Nam et al. (2007) explore the effects of service quality, modeling contract duration using a hazard rate model and usage with a Poisson regression model. Bolton and Lemon (1999) use a Tobit model to model usage of television entertainment and cellular communications services, finding a significant relationship between satisfaction and usage (Figure 1c).

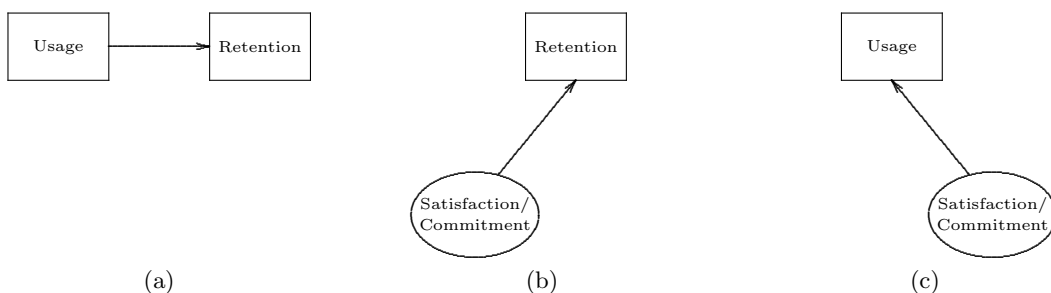


Figure 1: Existing modeling approaches

Although widely used in practice, none of the above methods can be used to address the modeling problem we are considering, either because they make use of survey data, or because the modeling of future behavior as a direct function of past behavior limits the forecast horizon. With regards to this latter point, such a modeling exercise sees the transaction database being split into two consecutive periods, with data from the second period used to create the dependent variable for the model (e.g., renew (yes/no) when modeling churn, number of transactions or total spend when modeling usage), while data from the first period are used to create the predictor variables. In many settings, period 1 behavior is frequently summarized in terms of each customers "RFM" characteristics: *recency* (time of most recent purchase), *frequency* (number of past purchases), and *mon-*

etary value (average purchase amount per transaction). Having calibrated the regression model, we can predict period 3 behavior using the observed period 2 data. However, it is difficult to use these models to forecast buyer behavior for period 4 when we are unable to specify values for the RFM predictor variables in period 3 for each customer; rather, we have to resort to simulated behavior. And so on. (See Fader et al. (2005) for a discussion of this and related problems with such a modeling approach.)

Despite the problems identified with this body of work (Figure 1), we can use the general findings to guide us as we develop our model. We propose a joint model of usage and retention under the assumption that both behaviors are driven by some unobserved characteristic, something that drives both phenomena and that can be inferred from them—Figure 2. This suggests that the observed association between usage and retention (Figure 1a) is in fact a spurious correlation. Some researchers explore the concept of “satisfaction” (Bolton et al. 2000, Bolton and Lemon 1999), while others talk of “commitment” (Gruen et al. 2000, Verhoef 2003). In this paper we recognize the existence of such an underlying construct and seek to model it, without formally defining it. Acknowledging the existence of this unobserved factor allows us to approach usage and retention processes jointly, and thus make accurate predictions of customer behavior in contractual settings. Furthermore, modeling the stochastic evolution of the underlying construct means that we can make predictions of behavior in future periods.

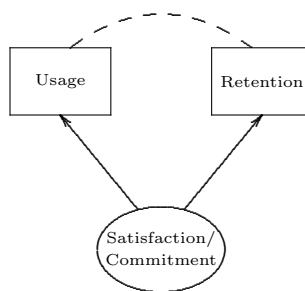


Figure 2: Proposed modeling approach

We propose a dynamic latent trait model in which usage and renewal behaviors are jointly driven by the same (individual-level) underlying process (Section 2). In Section 3 we validate the proposed model using data from a performing arts organization, and show how the proposed model can be used to provide additional insights of interest to managers.

We conclude with a summary of the methodological and practical contributions of this research, as well as a discussion of directions for future research (Section 4).

2 Model Development

Our goal is to develop a joint model of contract renewal and usage while “under contract”. We start by outlining the intuition behind our proposed model.

We assume that each customer’s *usage* (or consumption) and *renewal* (or churn) behavior reflects a latent trait that evolves over time. For example, let us consider a gym membership in which a customer pays a fee every month that gives her unlimited use of the sports facilities. In this example, the underlying trait can, for instance, be assumed to be a measure of “commitment to the gym”. This commitment is *reflected* in the number of times she goes to the gym each week (usage level). Furthermore, her monthly decision of whether or not to continue her membership reflects the latent trait; she will stop renewing her membership when her commitment is below a certain threshold. This is illustrated in Figure 3.

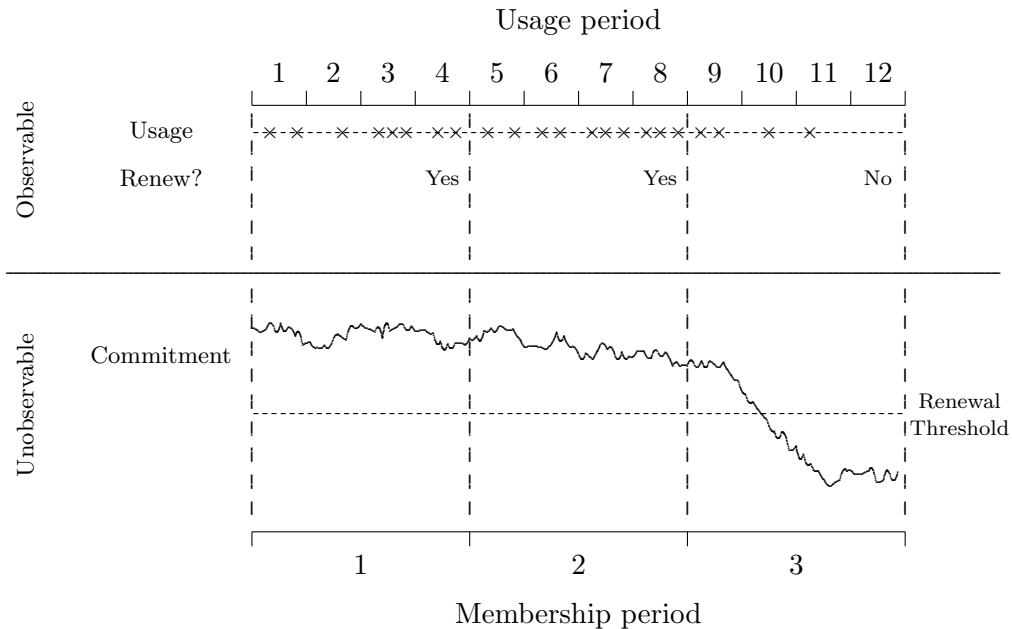


Figure 3: Model Intuition

More generally, we wish to model a bivariate process measured on different time scales;

in Figure 3, the time scale for the renewal process is four times that of the usage process. The model takes into account the individual dynamic latent process that drives the observable behaviors. Modeling the evolution of the latent trait will allow us to make simultaneous predictions about future usage and churn probabilities.

The proposed model must have three characteristics:

- i. It should handle bivariate data where one variable is binary (e.g., churn) while the other is not (e.g., usage).
- ii. The usage and renewal processes do not have the same “clock” (e.g., monthly usage vs. annual contract renewal, weekly attendance vs. monthly membership renewal).
- iii. It must be able to accommodate “informative” dropout. (The binary variable of interest (e.g., churn) is absorbing (i.e., it cannot transition from 0 to 1) and such “dropout” is “informative” about the usage process.)

Elaborating on the notion of absorbing dropout, the fact that an individual is active at a particular point in time implies that her underlying trait was above some renewal threshold in all preceding *renewal* periods. For example, with reference to Figure 3, the fact that this person is a member in the third month (periods 9–12) means that she renewed in periods 4 and 8. This tells us that her underlying trait had to be above the renewal threshold in usage periods 4 and 8. However, it does not tell us anything about the level of the latent variable in periods 1, 2, 3, 5, ...; this has to be inferred from her usage behavior.

2.1 Related Literature

A number of researchers have developed dynamic latent models. With few exceptions, they focus on linear models (also called linear state space models, or Dynamic Linear Models (DLM)), well suited for data generated by multivariate Gaussian processes (Gourieroux and Jasiak 2001, West and Harrison 1997, Van Heerde et al. 2004). However, the Gaussian assumption is not appropriate for the setting under study since the observed processes are count and binary variables. Thus, the first challenge we face when modeling this

phenomenon is that the normality assumption does not hold for any of the processes under study. While parametric models for dynamic count data have been proposed in the econometrics literature (Hausman et al. 1984, Brannas and Johansson 1996, Congdon 2003), these methods are intended to model univariate count data and are therefore not suitable for the bivariate process being considered here.

Other marketing researchers have proposed various models that capture consumers' evolving behavior. Sabavala and Morrison (1981), Fader et al. (2004) and Moe and Fader (2004a,b) present nonstationary probability models for media exposure, new product purchasing, and web site usage, respectively. Netzer et al. (2008) use a hidden Markov model to characterize the latent process that underlies individuals' donation behaviors. Lachaab et al. (2006) model preference evolution in a discrete choice setting using a random coefficients multinomial probit model in which the random coefficients are dynamic. A similar approach is used by Liechty et al. (2005) in a conjoint analysis setting. However none of these models accommodate observed dropout.

A number of biostatisticians have developed longitudinal latent models with "dropout" (e.g. Diggle and Kenward 1994, Henderson et al. 2000, Xu and Zeger 2001, Hashemi et al. 2003, Liu and Huang 2009). The general approach is to assume a latent process that is deteriorating over time, with dropout occurring when this underlying process crosses a certain threshold. The latent process behavior is estimated by using repeated observations of variables driven by this underlying structure. With variations particular to each problem, these models consist of a joint estimation of the measurement process and a survival function. However, given that in our setting renewal can only occur at certain points in time (monthly, quarterly, annually, etc.), while consumption is observed more frequently, it is possible to have individuals whose underlying commitment has been negative for some period/s but becomes positive before the next renewal opportunity. Thus, in contrast with all duration/survival models, being active in a certain period does not necessarily imply that the underlying trait was above zero for *every* preceding period, but only for those in which the renewal decision was made.

In conclusion, while the established longitudinal and latent-variable models do address each of the three required characteristics individually, none address all three simultane-

ously. We now turn our attention to the formal development of a model that does so.

2.2 Model Specification

Let t denote the usage time unit (periods) and i denote each customer ($i = 1, \dots, I$). For each customer i we have a total of T_i usage observations. Let n denote the number of usage periods associated with each contract period (e.g., if the usage unit of time considered is a quarter and the contract is annual, then $n = 4$).

The model comprises three processes, all occurring at the individual level:

- i. the underlying “commitment” process that evolves over time,
- ii. the renewal process that is observed only every n periods and takes the value 1 if a person renews, 0 otherwise, and
- iii. the usage process that is observed every period.

The Commitment Process

We assume that every individual has an underlying trait, which we will call “commitment”.² This underlying trait represents the predisposition of the customer to continue the relationship and to some extent, the predisposition to use the product/service provided. We allow this individual-level trait to change over time, and also assume that it is unobservable from the modeler’s perspective. In other words, we model “commitment” as a latent variable that follows a dynamic stochastic process.

In Figure 3, the latent trait is presented as evolving in continuous time. However, we model it as a discrete-time (hidden) Markov process. We assume that there exists a set of K states $\{1, 2, \dots, K\}$, with 1 corresponding to the lowest level of commitment and K to the highest. These states represent the possible commitment levels that each individual could occupy at any point in time. We assume that S_{it} , the state occupied by person i in period t , evolves over time following a Markov process with transition matrix $\Pi = \{\pi_{jk}\}$,

²We acknowledge that the concept “commitment” has been defined and previously studied in the marketing literature (e.g. Garbarino and Johnson 1999, Gruen et al. 2000, Morgan and Hunt 1994). Its theoretical definition and measurement is beyond the scope of this paper.

with $j, k \in \{1, \dots, K\}$. For the sake of model parsimony, we restrict the Markov chain to transitions between adjacent states. That is,

$$P(S_{it} = k | S_{it-1} = j) = \begin{cases} \pi_{jk} & k \in \{j-1, j, j+1\} \\ 0 & \text{otherwise.} \end{cases} \quad (1)$$

We also need to establish the initial conditions for the commitment state in period 1. We assume that the probability that customer i belongs to commitment state k at period 1 is determined by the vector $Q = \{q_1, \dots, q_K\}$, where

$$P(S_{i1} = k) = q_k, \quad k = 1, \dots, K. \quad (2)$$

Hidden Markov models (HMMs) were introduced in the marketing literature by Poulsen (1990) as a flexible framework for modeling brand choice behavior. Since then they have been applied in the marketing literature to model a wide range of behaviors (e.g., Montgomery et al. 2004; Moon et al. 2007; Smith et al. 2006; Netzer et al. 2008).

Netzer et al. (2008) use a HMM to capture customer relationship dynamics. The approach taken in the current study is similar to theirs in the sense that we also link transaction behavior to underlying customer relationship strength, in our case “commitment” level. However, our model specification differs from their approach in two ways. Firstly, they are working in a “noncontractual” setting (where attrition is unobserved) and thus map the latent states with just one observable behavior (i.e., transactions). In our setting customer attrition is observed, and therefore this information is used to define and identify the latent states. Secondly, they model a non-homogeneous transition process where the probability of switching among states is a function of the interactions between the firm and the customer. As such interactions do not occur in our empirical setting, we model the transition process in an homogeneous manner.

Having specified how the latent trait evolves over time, we now specify the mapping between this underlying construct and the two observable behaviors of interest, usage and renewal.

The Usage Process

While under contract, a customer’s usage behavior is observed every period. This behavior reflects her underlying commitment — for any given individual, we would expect higher commitment levels be reflected by higher usage levels. At the same time, we acknowledge that individuals may have different intrinsic levels of usage; in other words, unobserved cross-sectional heterogeneity in usage patterns. As such, our model should allow two customers with the same underlying pattern of commitment to have different usage patterns.

We propose two possible formulations for the usage process: Poisson and binomial. The Poisson process is the natural specification for modeling counts. Behaviors for which this specification is appropriate include the number of credit card transactions per month, the number of movies purchased each month in a pay-TV setting, and the number of phone calls made per week. However, in some settings the usage level has an upper bound, either because of capacity constraints from the company’s side, or because the time period in which usage is observed is short. For example, going back to the gym example, if one wants to model the number of days a member attends in a particular week, the Poisson may not be the most appropriate distribution since there is an upper bound of seven days. Similarly, consider the case of an orchestra wanting to predict the number of tickets that will be sold to their patrons. First, the number of performances attended is bounded by the total number of performances offered by the orchestra. Second, the number of performances offered should also be taken into consideration when predicting customers’ future attendances; there will be periods with higher demand simply because more performances are on offer and so the model should accommodate this information. It therefore makes sense to model usage using the binomial distribution. (We note that the binomial distribution can be approximated by a Poisson distribution for a high number of “trials” with a low probability of “success.”)

We first formalize the assumptions for the Poisson specification and then outline the changes that need to be made in order to accommodate the binomial specification. We assume that, for an individual in state k , the usage process (number of attendances,

transactions, visits, etc.) in period t follows a Poisson distribution with parameter

$$\lambda_{it} | [S_{it} = k] = \alpha_i \theta_k \quad (3)$$

where k is the (unobserved) commitment state of individual i at time t . In other words, the usage process is determined by a state dependent parameter θ_k that varies depending on the underlying level of commitment (which varies over time) and an individual specific parameter α_i that remains constant over time.

The parameter α_i captures heterogeneity in usage across the population, allowing two customers with the same commitment level to show different patterns of transactions. Individuals with higher values of α_i are expected, on average, to have a higher transaction propensity than those with lower values of α_i , regardless of their commitment level. The individual level parameter α_i is assumed to follow a gamma distribution with scale parameter r and a mean of 1.0).

The vector $\boldsymbol{\theta} = \{\theta_k\}$, $k = 1, \dots, K$ of state-specific parameters allows the customer's mean usage levels to change over time, as her underlying level of commitment changes. We impose the restriction that $\theta_k > 0 \forall k$ and is increasing with the level of commitment (i.e., $0 < \theta_1 < \theta_2 < \dots < \theta_K$). Notice that for each individual i , the expected level of usage is increasing with her commitment level, and even in the lowest commitment state, we can still observe non-zero usage.

Let $\tilde{S}_i = [S_{i1}, S_{i2}, \dots, S_{iT_i}]$ denote the (unobserved) sequence of states to which customer i belongs during her entire lifetime, with realization $\tilde{s}_i = [s_{i1}, s_{i2}, \dots, s_{iT_i}]$, where s_{it} takes on the value $k = 1, \dots, K$. The customer's usage likelihood function is

$$\begin{aligned} L_i^{\text{usage}}(\boldsymbol{\theta}, \alpha_i | \tilde{S}_i = \tilde{s}_i, \text{data}) &= \prod_{t=1}^{T_i} P(Y_{it} = y_{it} | S_{it} = k, \boldsymbol{\theta}, \alpha_i) \\ &= \prod_{t=1}^{T_i} \frac{e^{-\alpha_i \theta_k} (\alpha_i \theta_k)^{y_{it}}}{y_{it}!}. \end{aligned} \quad (4)$$

where y_{it} is customer i 's observed usage in period t .

Turning to the binomial specification, we let m_t denote the number of transaction

opportunities (e.g., number of performances offered, number of days in a particular period of time) and p_{it} the probability of a transaction occurring at any given transaction opportunity for customer i in period t . As with the Poisson specification, the transaction probability depends on the individual specific time-invariant parameter α_i and the commitment state at every period:

$$p_{it} | [S_{it} = k] = \theta_k^{\alpha_i}. \quad (5)$$

This specification also guaranties that the transaction probability is increasing with the level of commitment. The usage propensity parameter α_i is also assumed to follow a gamma distribution with equal scale parameter r and a mean of 1.0.

We impose the restrictions that $0 < \theta_k < 1$ for all k and that they increase with the level of commitment (i.e., $0 < \theta_1 < \theta_2 < \dots < \theta_K < 1$). The inclusion of α_i as an exponent (as opposed to a multiplier) ensures that the transaction probabilities remain bounded between zero and one.³

It follows that the customer's usage likelihood function is

$$\begin{aligned} L_i^{\text{usage}}(\boldsymbol{\theta}, \alpha_i | \tilde{S}_i = \tilde{s}_i, \text{data}) &= \prod_{t=1}^{T_i} P(Y_{it} = y_{it} | S_{it} = k, \boldsymbol{\theta}, \alpha_i, m_t) \\ &= \prod_{t=1}^{T_i} \binom{m_t}{y_{it}} (\theta_k^{\alpha_i})^{y_{it}} (1 - \theta_k^{\alpha_i})^{m_t - y_{it}}. \end{aligned} \quad (6)$$

The Renewal Process

At the end of each contract period (i.e., when $t = n, 2n, 3n, \dots$), each customer decides whether or not to renew her contract for the following n periods based on her current level of commitment. We assume that a customer does not renew if her commitment state is 1 (the lowest commitment level); otherwise she renews. Given that in period 1 all customers have freely decided to take out a service contract, we restrict the commitment state to be different from 1 in the first period (i.e., $q_1 = 0$). If a customer is active in a given

³Since this transformation is not linear in α_i , the average probability of transaction across all customers belonging to state k is not equal to θ_k ; this quantity is found by taking the expectation of $\theta_k^{\alpha_i}$ over the distribution of α_i .

period t , her commitment state in all preceding renewal periods $\tau = n, 2n, \dots$, with $\tau \leq t$, had to be different from 1; otherwise she would not have renewed her contract and been active at time t . However, an active customer could have been in state 1 in any preceding non-renewal period (i.e., $t \neq n, 2n, \dots$).

For example, let us consider a gym membership that is renewed monthly and where we observe individual attendances on a weekly basis. While usage is observed at every week, renewal/non-renewal can only happen at week 4 (end of first month), week 8 (end of second month), etc. Therefore, the fact that an individual is active in a particular month implies that her commitment level *at the end* of all preceding months (i.e., weeks 4, 8, ...) was different from 1. Figure 1 shows examples of sequences of commitment states that, based on our assumption of the renewal process, can or cannot occur in our setting:

$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$	$t = 6$	$t = 7$	$t = 8$	$t = 9$	
1	3	1	2	2	3	2	2	3	✗
2	3	1	1	2	3	2	2	3	✗
2	3	2	2	2	3	2	1	3	✗
2	3	1	1	–	–	–	–	–	✓
2	3	2	2	2	3	2	1	–	✓
2	3	2	2	2	3	2	2	3	✓
2	1	1	2	2	1	2	2	1	✓

Table 1: Hypothetical sequences of commitment states

The first sequence of states cannot occur since, for an individual to have become a customer, her commitment in period 1 is by definition different from 1. The following two sequences of states can also not occur because if a customer is active in month 3 (week 9), her commitment at the end of months 1 and 2 (weeks 4 and 8) had to be greater than 1. Notice that there is no restriction about her commitment in periods other than 4 and 8, thus the last sequence shown in the table can occur.

Bringing It All Together

Now that the renewal processes has been specified, we need to combine it with the sub-model for usage behavior to characterize the overall model.

For each customer i , we have shown how the unobserved sequence \tilde{S}_i determines her

renewal pattern over time. Moreover, conditional on her $\tilde{S}_i = \tilde{s}_i$, the expression for the usage likelihood was derived for both the Poisson and binomial specifications. To remove the conditioning on \tilde{s}_i , we need to consider all possible paths that \tilde{S}_i may take, weighting each usage likelihood by the probability of that path:

$$L_i(\alpha_i, \boldsymbol{\theta}, \Pi, Q | \text{data}) = \sum_{\tilde{s}_i \in \Upsilon} L_i^{\text{usage}}(\boldsymbol{\theta}, \alpha_i | \tilde{S}_i = \tilde{s}_i, \text{data}) f(\tilde{s}_i | \Pi, Q), \quad (7)$$

where Υ denotes all possible commitment state paths customer i might have during her lifetime, $L_i^{\text{usage}}(\boldsymbol{\theta}, \alpha_i | \tilde{S}_i = \tilde{s}_i, \text{data})$ is substituted by (4) or (6) depending on whether we estimate the Poisson or the binomial model, and $f(\tilde{s}_i | \Pi, Q)$ is the probability of path \tilde{s}_i happening.

If there were no restrictions due to the renewal process, the space Υ would include all possible combinations of the K states across T_i periods (i.e., K^{T_i} possible paths). However, as discussed earlier, the nature of the renewal process places constraints on the underlying commitment process. As a consequence, Υ contains $(K - 1)^{\lfloor (T_i - 1)/n \rfloor} K^{T_i - \lfloor (T_i - 1)/n \rfloor - 1}$ possible paths.

Considering all customers in our sample, and recognizing the random nature of α_i , the overall likelihood function is:

$$L(\boldsymbol{\theta}, \Pi, Q, r | \text{data}) = \prod_{i=1}^I \int_0^\infty L_i(\alpha_i, \boldsymbol{\theta}, \Pi, Q | \text{data}) f(\alpha_i | r) d\alpha_i. \quad (8)$$

In summary, we have proposed a hidden Markov model combined with a heterogeneous Poisson or binomial process to model bivariate data where the two processes occur on different time scales. The hidden Markov process captures dynamics at the individual level as well as renewal behavior, while the Poisson or binomial process links these underlying dynamics with usage behavior allowing for unobserved individual heterogeneity. The resulting model has $(3K - 2) + (K - 1) + K + 1$ population parameters, which are the elements of Π , Q , $\boldsymbol{\theta}$ and r , respectively. We estimate these model parameters using a hierarchical Bayes framework. In particular, we use data augmentation techniques to draw from the distribution of the latent states S_{it} as well as the individual-level parameter α_i .

We control for the path restrictions (due to the nature of the contract renewal process) when augmenting the latent states. As a consequence the evaluation of the likelihood function becomes simpler, reduced to the expression of the conditional (usage) likelihood function, $L_i^{\text{usage}}(\boldsymbol{\theta}, \alpha_i | \tilde{S}_i = \tilde{s}_i, \text{data})$. See Appendix A for details.

3 Empirical Analysis

3.1 Data

We explore the performance of the proposed model using data from a European performing arts organization. This organization runs a so-called Friends scheme. An annual membership of this scheme provides “Friends” with several non-pecuniary benefits, including priority ticket booking,⁴ newsletters, and invitations to special events.

In addition to the membership fee, Friends are an important source of income for the organization through their buying of tickets. The company generates approximately \$5 million a year from membership fees alone and a further \$40 million from members’ bookings. Each year is divided into four booking periods; all members receive a magazine each booking period with information about the performances offered in the next period and a booking form to purchase tickets. Given that two performances cannot be conducted at the same time, the number of performances offered during each booking period is limited. Furthermore, some periods see more matinee performances being offered than others, implying that the number of available performances changes slightly from one booking period to the other. When one’s Friends membership is close to expiring (generally one month before the cancellation date), the company sends out a renewal letter. If membership is not renewed, the benefits can no longer be received.

This organization offers five different types of membership that vary by price and benefits received. In this paper we focus on those individuals who have taken out the lowest level of membership; this accounts for over 80% of the entire membership base. We focus on the cohort of individuals who took out their first Friends membership during the

⁴In contrast to the subscription schemes associated with many North American performing arts organizations, tickets are not included as part of the scheme.

first quarter of 2002, and analyze their renewal and booking behaviors for the following 4 years (16 booking periods). Among the 1,173 members that meet these criteria, 884 renewed in year 2 (26.6% churn), 738 renewed in year 3 (16.5% churn), 634 renewed at least three times (14.1% churn) and 575 members were still active after the four years of observation. Expressing these data in terms of periods (as we defined t in section 2.2), we have a total of 17 periods. We observe usage in periods 1 to 16 and renewal decisions in periods 5, 9, 13, and 17.⁵

This cohort of customers made a total of 14,255 bookings across the entire observation period. On average, a member makes 1.05 bookings per booking period. However the transaction behavior is very heterogeneous across members, with the average number of bookings per booking period ranging from 0 to 41.9. (Given the nature of these data, the words *booking* and *transaction* are used interchangeably.) Figure 4 shows the total number of transactions (in bars) and the number of active members (dotted line) for each booking period. We observe that total number transactions decreases over time, mostly due to membership cancellations.

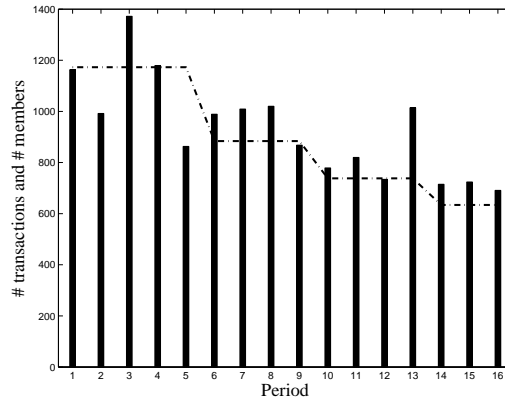


Figure 4: Observed usage and renewal behavior

⁵In developing the logic of our model, we discussed a contract period of $n = 4$ with renewal occurring at 4, 8, 12, etc. This implicitly assumed that customers are acquired immediately at the beginning of the period (e.g. January 1, the first day of Q1) with the contract expiring at the end of fourth period (e.g., December 31, the last day of Q4.), when the periods are quarters. In this empirical setting, customers are acquired throughout the first period, which means the first renewal occurs sometime in the fifth period.

3.2 Model Estimation and Results

We split the four years of data into a calibration period (periods 1 to 11) and a validation period (periods 12 to 17). Note that, for this cohort, renewal decisions are made in periods 5, 9, 13, and 17. We can therefore examine model performance in the validation period in three ways. First, we will examine how well the model predicts usage for the remaining periods under the current contract (i.e., forecast usage before the membership expires). Second, we will examine the accuracy of the model’s predictions of renewal in all future *renewal* periods (i.e., at the end of the current contract and at the end of following contract). Third, we will examine the accuracy of the model’s predictions of usage *conditional* on renewal (i.e., for those customers for which the model predicts renewal, we forecast usage in future periods).

We first need to determine the most appropriate model (Poisson vs. binomial usage process) and the optimal number of states to be considered in the (hidden) Markov chain. Given the nature of this empirical setting, where there is a limited number of performances on offer during each booking period and this number changes across periods, we focus on the binomial specification for the usage process.⁶ We estimate the model varying the number of (hidden) states from 2 to 5, and compute: (i) the marginal log-density,⁷ (ii) the Akaike information criterion (AIC), (iii) the logarithm of the Bayes factors, and (iv) the in-sample Mean Absolute Percentage Error (MAPE) in the predicted number of transactions.

As shown in Table 2, the specification with the highest marginal log-density and AIC values is the model with 4 hidden states. The Bayes factor of this specification, compared with a more parsimonious model also gives support to the 4 state model. Regarding the in-sample predictions, we also find that the model with 4 states is marginally the most accurate, with an MAPE of 9.63%.

⁶Since the number of performances offered per booking period (m_t) is high, we would expect the two models to be quite similar in their results. Fitting the Poisson specification, we find that this is the case.

⁷The marginal log-density is computed using the harmonic mean on the likelihoods across iterations (Newton and Raftery 1994).

# States	Marginal	AIC	Log	MAPE
	Log-density		Bayes Factor	
2	-11,901.5	23,357.1	-	13.58
3	-11,131.5	21,781.1	770.0	9.66
4	-11,111.5	21,283.3	20.0	9.63
5	-11,262.2	22,236.8	-150.7	9.66

Table 2: Model selection criteria

Model Results

Table 3 presents the posterior means and intervals for the parameters of the usage model under the binomial specification with 4 states.

	Parameter	Posterior mean	95% Interval
Commitment parameters	θ_1	0.0016	[0.0013 0.0019]
	θ_2	0.0017	[0.0014 0.0020]
	θ_3	0.0066	[0.0059 0.0077]
	θ_4	0.0512	[0.0394 0.0694]
Heterogeneity	r	21.830	[19.520 24.350]

Table 3: Parameters of the binomial usage model with 4 states

The first set of parameters (θ_i s) correspond to the usage parameter common to all customers belonging to a particular commitment state. The r parameter measures the degree of unobserved heterogeneity in usage behavior within each state. Recalling (5), the distributions of the transaction probabilities for all individuals in each commitment state are reported in Figure 5.⁸

The mean transaction probability for state 1 is 0.0035 and for state 2 is 0.0037. The important difference between these two states is with regards to renewal behavior. The interpretation of each state is determined by the transaction propensity and the renewal behavior. Hence, while those individuals in state 1 will on average make ever-so-slightly fewer transactions than those in state 2, they will churn if they remain in that state during a renewal period. On the contrary, individuals in state 2 will renew their membership even though they also have low transaction propensities. For individuals in state 3, the

⁸The exact expression for the pdf is $f(p_{it} | \theta_k, r, S_{it} = k) = \frac{r^r \exp(-r \ln(p_{it}) / \ln(\theta_k))}{p_{it} \Gamma(r) |\ln(\theta_k)|} \left[\frac{\ln(p_{it})}{\ln(\theta_k)} \right]^{r-1}$.

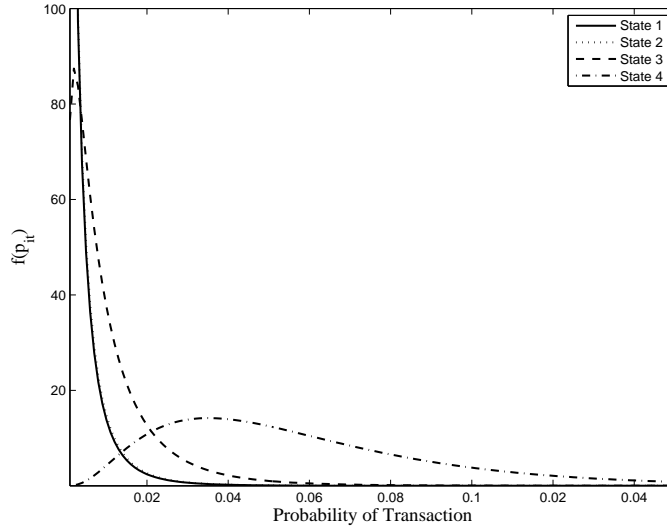


Figure 5: Distributions of transaction probabilities

average probability of transaction is 0.011, approximately three times greater than those associated with states 1 and 2. This translates to an average of 1.87 transactions per booking period (more than 7 transactions in a year) for those members in state 3, while customers in states 1 and 2 are expected to make an average of 0.59 and 0.62 transactions per booking period, respectively. The highest commitment level (state 4) corresponds to the very active members in terms of usage behavior, with an average of 10.6 transactions per booking period.

Dynamics in the underlying trait are captured by the hidden Markov chain. Table 4 shows the posterior means of the transition probabilities, while Table 5 reports the estimates of Q , the initial conditions for the commitment state in period 1. We observe a very dynamic pattern in this setting. With the exception of state 1, the probability of switching out of a state is high. For instance, for a customer in state 2, the probability of switching to state 3 in a particular period is 0.655, whereas the probability of switching to the lowest commitment state (1) is 0.172. We note that state 1 is not an absorbing state (i.e., $P(S_{it} = 1 | S_{i,t-1} = 1) \neq 1$). This is due to the fact that not all periods are renewal occasions. Therefore, even though all members in state 1 will not renew if they are in a renewal period, it is possible to find individuals who were in state 1 at a particular time and changed their commitment state before the renewal occasion occurred.

From state	To state			
	1	2	3	4
1	0.903	0.097	–	–
2	0.172	0.169	0.655	–
3	–	0.427	0.567	0.006
4	–	–	0.434	0.566

Table 4: Estimated transition probabilities

With reference to Table 5, the q_s correspond to the probabilities of being in each state when an individual first joined the membership scheme. This is the distribution of underlying states for a just-acquired member of this cohort. The probability of a newly acquired member being in the highest state is very low; just over 1% of the members acquired in 2002 were deemed to be in the top commitment state in period 1. Most members were distributed between states 2 and 3 (41% and 58% respectively) when they joined the Friends scheme.

Parameter	Posterior mean	95% Interval
q_2	0.410	[0.132 0.703]
q_3	0.578	[0.282 0.856]
q_4	0.011	[0.001 0.028]

Table 5: Estimated period 1 state probabilities

In-sample Fit

Figure 6 shows actual and predicted levels of usage for the in-sample period. The figure shows an excellent fit of the proposed model when predicting total usage. The MAPE for the overall calibration period is 9.63%.

In addition to being able to track aggregate levels of usage, it is important that the model be able to capture cross-sectional heterogeneity. We compute the distribution of the number of attendances at the individual level. We compare the maximum, the minimum, and five common percentiles of this distribution with the means for their respective predictive distributions. Table 6 shows how all measures lie in the 95% confidence interval of the mean posterior distributions.

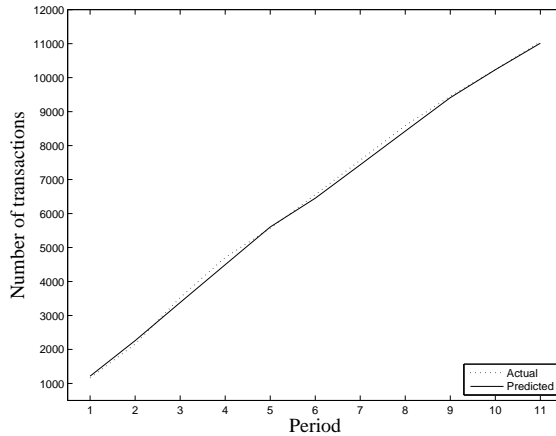


Figure 6: Usage (in-sample) predictions

	Actual	Posterior mean	95% Post. interval
Min	0.00	0.00	[0.00 0.00]
5%	0.00	0.00	[0.00 0.00]
25%	2.00	1.81	[1.00 2.00]
50%	4.00	4.59	[4.00 5.00]
75%	11.00	11.07	[10.00 12.00]
95%	32.85	32.80	[31.00 355.00]
Max	457.00	444.87	[397.00 492.00]

Table 6: Distribution of in-sample predictions

3.3 Forecasting Performance

Having fitted the model to the calibration period data, we will examine the performance of the model in the hold-out validation period. In addition to comparing its performance relative to a set of benchmark models, we also consider two restricted versions of the proposed model: a static latent trait model ($\Pi = I_K$) in which there is heterogeneity in transaction behavior but members cannot change their commitment state over time, and a dynamic latent trait model with homogeneous transaction behavior ($r \rightarrow \infty$) in which all members in each commitment state have the same expected transaction behavior.⁹

We first forecast usage behavior in periods 12 and 13 for all members that were active at the end of our calibration period. Then, conditional on each individual’s underlying state in period 13, we predict renewal behavior at that particular moment. Finally, conditional

⁹The marginal log-densities of these two models are $-12,341.5$ and $-12,831.5$, respectively.

on having renewed at that time, we forecast usage behavior for all remaining periods and renewal behavior for the last period of data. This time-split structure allows us to analyze separately *usage* forecast accuracy (comparing actual vs. predicted number of transactions in period 12), *renewal* forecast accuracy (comparing actual and predicted renewal rates in periods 13 and 17) and *overall* forecast accuracy (comparing actual and predicted usage levels in periods 14 onwards).

Usage Process

In order to assess the quality of the usage predictions, we compare the forecasts from the proposed model (both the full and two restricted versions) with those generated using two RFM-based negative binomial (NB) regression models — see Appendix B for details — and two heuristics (drawing on the work of Wübben and Wangenheim (2008)). Heuristic A, periodic usage, assumes that each individual repeats the same pattern every year. Heuristic B, status quo, assumes that all customers will make as many transactions as their current average.¹⁰

To assess the validity of the usage predictions, we compare the models’ forecasts in period 12. (We cannot generate a forecast for period 13 (and beyond) using the RFM models because the RFM characteristics are not available for future periods.) The predictive performance is compared at the aggregate level, looking at the percentage error (PE) in the predicted total number of transactions, as well as at disaggregate level, looking at the histogram of the population distribution of the number of transactions. That is, we compute, based on the model predictions, how many customers have zero transactions, one transaction, two transactions, etc. and compare these values with the actual data. In order to make the histograms comparable, we compute the Chi-square goodness of fit (χ^2) statistic for all models. This is a measure of how well the model recovers the distribution of transactions across the population; the smaller the χ^2 statistic, the better fit (i.e., the more similar the distributions of actual and predicted number of transactions).

¹⁰For example, suppose a customer makes 2, 4, 2, 4 transactions over the preceding four periods. Under heuristic A, we would predict that this customer will make 2, 4, 2, 4 transactions over the next four periods. Under heuristic B, we would predict a pattern of 3, 3, 3, 3.

Table 7 shows the error measures for all usage models. Considering both measures of fit, the proposed model outperforms all other methods. The aggregate level predictions are very good for all specifications of the proposed model (PE < 5%), with the static specification of the proposed method having the smallest forecast error. Although it may seem that the static specification gives better estimates of future usage, when considering the fit at the distribution level, we observe that the full (dynamic heterogeneous) specification recovers the distribution of number of transactions more accurately. The χ^2 of the proposed model is 16.4, whereas the static and homogeneous specifications give χ^2 values of 28.6 and 48.9, respectively. To better understand the implications of having a higher χ^2 (in other words worse disaggregate measure of fit) we show in Figure 7 the histograms of the number of transactions predicted by the best three methods on the basis of the aggregate predictions. The first column corresponds to the number of members who did not make any transactions, the second column represents the number of customers who made one transaction and so forth. We observe that the static specification underpredicts the number of customers who did not make any transactions and overestimates the number of customers who made one transaction. Even though the proposed method provides a slightly higher PE at the aggregate level, these histograms show that it predicts usage more accurately than any of the other methods.

	Aggregate % error	Disaggregate χ^2
Heuristic		
A (periodic)	28.8	20.8
B (status quo)	21.8	147.9
NB regression		
Cross-sectional	9.5	264.7
Panel	14.9	21.8
Latent trait		
Static	-2.0	28.6
Homogeneous	-3.9	48.9
Full	-4.3	16.4

Table 7: Predictive performance for usage behavior in period 12

It is worth noting that, based on the results presented in Table 7, one might think

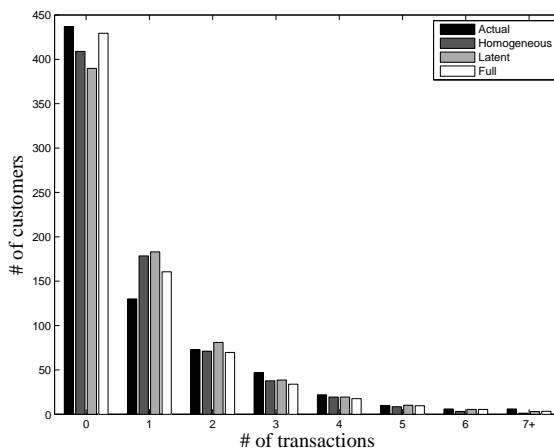


Figure 7: Distribution of (predicted vs. actual) number of transactions in period 12

that heuristic A is not a bad model because its χ^2 is low. However, this should not be surprising given that heuristic A predicts that every customer will make exactly the same number of transactions as the same period last year. In other words, by construction, this method replicates the population distribution year after year. Nevertheless, if we look at the error of aggregate number transactions, we see that the heuristic A method is not accurate in its predictions, over-forecasting the total number of bookings by 28.8%.

Renewal Process

In order to assess the quality of the churn predictions, we compare the predicted renewal rates of the proposed model (both the full and two restricted versions) with those generated using two RFM-based logistic regression models—see Appendix B for details—and two heuristics. The first heuristic (C) says that churn occurs if there is no usage activity during the last two periods (Wübben and Wangenheim 2008). The second heuristic (D) says that churn occurs if an individual’s average usage over the last two periods is lower than that of the corresponding periods in the previous year. (This is in the spirit of Berry and Linoff’s (2004) discussion of how changes in usage can be a leading indicator of churn.)

The predictive performance for all churn models is presented in Table 8. We compare actual versus predicted renewal rate in periods 13 and 17. As shown in the table, the proposed model provides the most accurate predictions of the future renewal rate. Both heuristic methods provide very poor estimates for future churn. The two logistic regression

models overestimate future renewal (and therefore the size of the customer base) by more than ten percentage points. The static and homogeneous specifications of the proposed model underestimate renewal by 73.7% and 7.7%, respectively. In all cases, the absolute error is notably higher than that associated with the full specification of the proposed model (with a percentage error of -6.0%).

	Period 13		Period 17	
	Renewal Rate	% error	Renewal Rate	% error
Heuristic				
C	27%	-68.8	-	-
D	63%	-26.5	-	-
Logistic regression				
Cross-sectional	97%	12.8	-	-
Panel	97%	12.9	-	-
Latent Trait				
Static	23%	-73.7	-	-
Homogeneous	79%	-7.7	80%	-12.2
Full	81%	-6.0	81%	-10.8
Actual	86%	-	91%	-

Table 8: Actual and predicted renewal rates

We cannot use the first four methods/models to generate forecasts of churn in future periods (e.g., period 17) since they are based on measures of usage behavior which are unobserved for future periods. (For example, in order to use the logistic regression model to predict period 17 churn, we would need to know how many transactions each customer made up-to-and-including period 16.) Additionally, the static class model cannot be used to predict renewal in period 17 because this specification does not allow customers to change their commitment level over time. Although the homogeneous specification of the proposed model provides a good prediction, the full specification provides the most accurate forecasts of future renewal behavior.

Renewal and Usage Processes

Finally, we consider the overall forecasting accuracy of the models, comparing actual and predicted usage levels in periods 14 onwards. One problem we encounter with the benchmark models is that none of them can be used to generate a forecast of customer

behavior over the entire validation period, either because they cannot predict behavior beyond the next period or because they do not account for the attrition process. However, if we are interested in forecasting customer behavior in contractual settings, we need a model that accounts for both usage and churn predictions. Aside from the full specification of the proposed model, the two restricted versions of the proposed model (static and homogeneous model) also account for both processes. Additionally, we use a combination of the most accurate churn benchmark model (panel logistic regression) with the best heuristic usage model (status quo) in the following manner.¹¹ We first predict renewal behavior in period 13 using the panel logistic regression. Then, for those individuals who are predicted to renew, we use Heuristic A to forecast usage. As before, we examine the accuracy of the predictions at the aggregate and disaggregate level.

We examine usage behavior in periods 14 to 16, which in turn depends on predicted renewal behavior in period 13. We compare the MAPE across all forecast periods (Table 9) and find that the proposed model gives the most accurate predictions over the entire validation period (MAPE=10.9%). The second-best method is the homogeneous specification, with an MAPE of 11.7%. (In other words, a model that does not account for unobserved usage heterogeneity within segments increases the forecast error by 7.7%.) The static specification provides the biggest error predictions with an MAPE of 23.7%, more than double the error associated with the full specification. The predictions associated with the combined benchmark models are also poor, with an MAPE of 20.9%. This result reinforces the suggestion that the inclusion of dynamics yields more accurate predictions. Similar results are obtained when considering the disaggregate measures of fit (average χ^2 across the three forecast periods); the full specification of the proposed model is clearly superior.

In summary, we have shown that the full specification of the proposed model predicts retention and usage behavior accurately. Furthermore, it outperforms benchmark models on both the usage and retention dimensions.

¹¹We cannot use any of the usage regression models as they need as-yet unobserved RFM data in order to make predictions for periods 14–16.

	Out-of-sample MAPE	(Average) χ^2
Regression + Heuristic	20.9	274.7
Latent Trait		
Static	23.7	210.7
Homogeneous	11.7	22.4
Full	10.9	16.3

Table 9: Predictive performance for both processes

3.4 Additional Results

In addition to predicting future usage and retention behaviors, this model also provides several insights that can help the marketer better understand her customer base. In particular, this model allows us to segment customers dynamically on the basis of their underlying commitment levels, which can guide retention efforts. Moreover, the model not only enables us to detect at-risk customers (potential churners), but also to identify highly committed customers. This is in the marketer’s interests if, for instance, the organization wants to target specific supporters in the course of its fundraising activities.

Recovering State Membership

The hidden Markov specification allows us to dynamically segment the customer base given usage and renewal behavior.¹² In contrast to the static model, the hidden Markov model provides insights into the underlying behavioral dynamics of the customer base. To do so, we need to compute the number of customers in each commitment state and track this information over time. Recovering the state membership is straightforward when using data augmentation techniques to estimate the hidden states. In each period we can easily compute the number of customers in each segment, and look at how the segment sizes evolve over time. Figure 8a shows the size of each hidden state for all periods; the numbers for periods 12–17 are forecasts. We observe that the size of state 1 (bottom black) increases over time and then radically drops after periods 5, 9 and 13. This is due to the churn process; based on our model assumptions, all customers in state 1 in the

¹²Hereafter we will use the terms segment and state interchangeably.

renewal period do not renew their membership. Consequently, the total height of the bars also decreases after the renewal periods.

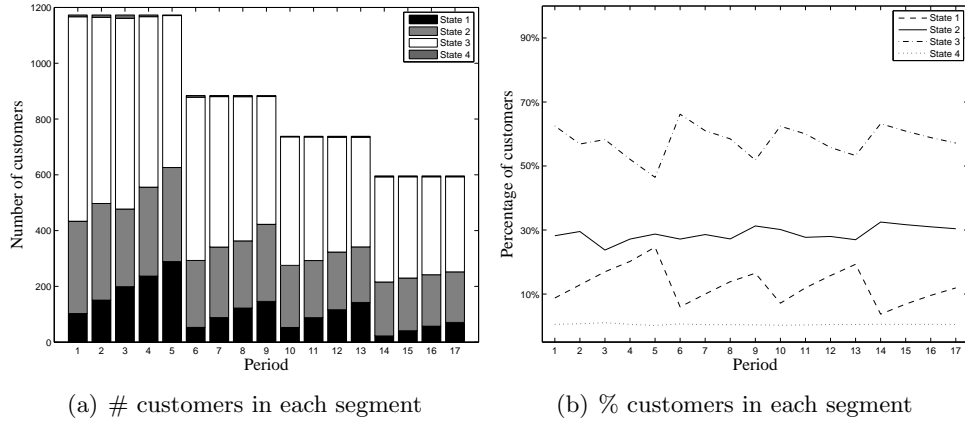


Figure 8: Segment dynamics

Figure 8b shows the share of each state (i.e., percentage of active customers in each segment). We observe that state 2's share is fairly stable, accounting for approximately 30% of the customer base. This does not imply that the same customers belong to state 2 in all periods; the volatility we observe in states 1 and 3 (Figure 8b) and the parameters of the transition matrix (Table 4) mean that in every period, many customers switch from state 3 to 2, and from state 2 to 1, but not necessarily the same individuals. In order to analyze the individual-level transitions, we must recover the distribution of the states membership for each customer over her lifetime. Then, looking at the posterior probability of belonging to each state, we can analyze the commitment dynamics at the individual level.

As an illustration, we analyze the evolution of state membership for three customers who cancelled their subscription after two years (in period 9). Figure 9 shows how the distribution of state membership varies during the year prior to their cancellation. We observe that customers A and B have a similar "commitment" pattern: at the beginning of the second year (period 5), both customers are equally likely to belong to states 2 and 3. Their "commitment" level seems to increase in period 6 and then decreases in periods 7 and 8, when the probability of staying in state 1 becomes larger, almost 0.8 in period 8. The state history for customer C is slightly different. Not only was this customer more

likely to belong to state 2 at the beginning of the second year, her probability of being state 1 (lowest commitment level) also increased very quickly.

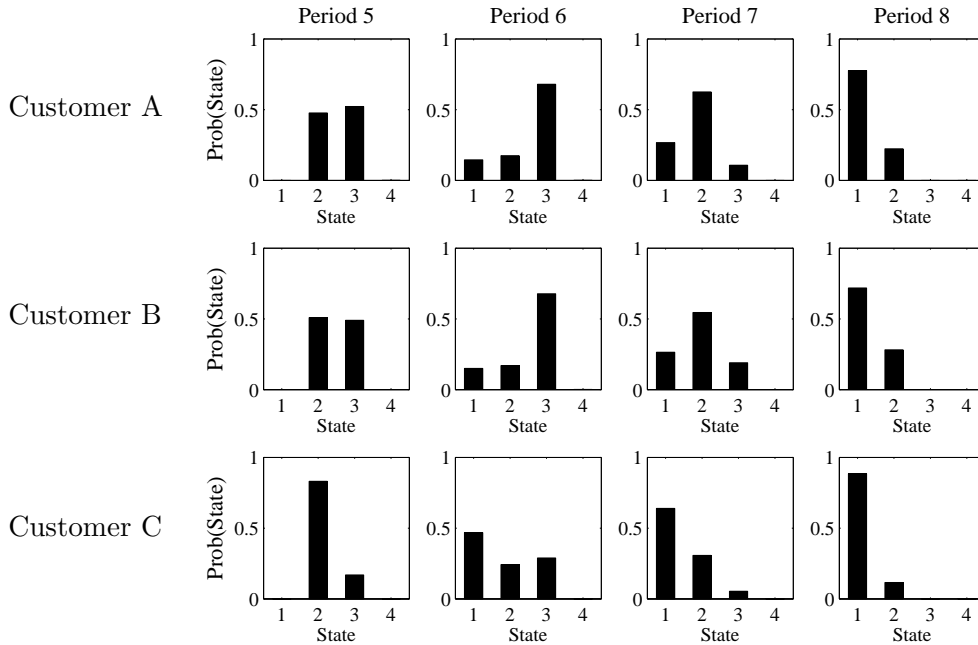


Figure 9: Individual-level dynamics

We can contrast this information with the actual transaction behavior to better understand the dynamics of the model. Table 10 shows the observed transaction pattern of these three customers. Consistent with the state evolution shown in Figure 9, we observe that customer C’s usage dropped very early in the second year of membership, while customer A kept her average level of usage up to period 6. We note that even though customers A and B had almost identical state histories, their observed usage behavior is very different. Moreover, while customers B and C have exactly the same transaction behavior during the second year of membership, their underlying “commitment” history is very different because of their different transaction patterns during the first year of membership.

Recovering the underlying states over time allows us to identify those customers who are likely to have changed (decreased or increased) their commitment state recently. This useful piece of information would help the marketer target customers differently. For example, in our setting, the organization is interested in knowing, before the cancellation date, which members have recently suffered a drop in their underlying commitment level,

	Year 1				Year 2			
	1	2	3	4	5	6	7	8
Customer A	2	1	0	1	1	2	0	1
Customer B	0	0	0	0	0	1	0	0
Customer C	2	1	4	3	0	1	0	0

Table 10: Customer-level usage behavior (actual)

so that pre-emptive retention activities can be undertaken. (As illustrated by the poor performance of Heuristics C and D in predicting churn, such at-risk customers cannot be identified without the use of a formal model.)

Incorporating New Information

In dynamic environments, updating the list of at-risk customers is central to any efforts designed to increase the success of retention campaigns. Apart from using past behavior to identify changes in commitment level (as illustrated in Figure 9), another way to detect changes in commitment levels is to use the most recent piece of information, when available, to update the model estimates. For example, having estimated the model using data from periods 1–11, suppose we now observe usage behavior for period 12. What can we learn from this additional information? In other words, how can this new data be used to update what we knew already about the customer base? Once the model is calibrated, it is easy to incorporate new information as more behavior is observed. We can simply use the model estimates at period 11 and use Bayes’ rule to update the parameter estimates given the individuals’ period 12 transaction data. More specifically, using the posterior distribution of state membership at period 11 ($S_{i,11}$) and the estimated transition matrix (Π), we compute the prior distribution of state membership in the following period (i.e., $P(S_{i,12} = k|S_{i,11}, \Pi)$ for $k = 1, \dots, K$). Then, we calculate the likelihood of observing period 12 usage behavior conditional on the individual-level parameters α_i . Finally, we compute the posterior distribution of state membership in period 12 using Bayes’ rule:

$$P(S_{i,12} = k|Y_{i,12} = y_{i,12}, \mathbf{\Omega}) = \frac{P(Y_{i,12} = y_{i,12}|S_{i,12} = k, \alpha_i, \boldsymbol{\theta})P(S_{i,12} = k|S_{i,11}, \Pi)}{\sum_{l=1}^K P(Y_{i,12} = y_{i,12}|S_{i,12} = l, \alpha_i, \boldsymbol{\theta})P(S_{i,12} = l|S_{i,11}, \Pi)} \quad (9)$$

where Ω denotes all the model parameters.

We can now compare the distribution of state membership in periods 11 and 12 and identify those customers who have changed their commitment state in the current period. Following this procedure, we identify 45 customers who have shifted in their commitment level to the lowest state from period 11 to period 12.¹³ Similarly, we identify 2 members who moved to state 4 (highly committed level) in period 12. The main advantage of the updating process is that it requires simple computations and can serve as the basis for a dynamic resource allocation tool.

Moreover, the newly observed usage behavior can be used to update the model forecasts and generate more accurate predictions of future customer behavior. For instance, we can update using the period 13 usage information, and re-estimate the renewal rate at the end of the same period (i.e., the proportion of customers in state 1). Table 11 shows the improvement in renewal forecasts, compared with those generated at the end of the calibration sample (as shown in Section 3.3). Forecasts get more accurate as more information is added.

	Renewal Rate	% error
Calibration sample	81%	-5.99
Calibration + period 12 usage	82%	-5.12
Calibration + periods 12 and 13 usage	83%	-3.89
Actual	86%	-

Table 11: Renewal predictions with updated usage information

In summary, we have shown how the model can be used as a dynamic segmentation tool that enables us to detect potential churners as well as highly committed customers in every period. Once the model is calibrated, it is simple to incorporate new information as more behavior is observed, allowing the marketer to update the information about the customer base without major computational effort.

¹³We determine each customer's commitment level by selecting the state with the maximum probability.

External Validity

We have developed a joint model for usage and renewal behavior in contractual settings. The model incorporates both behaviors for two reasons: first, these are the two key drivers of customer profitability and, second, this information is generally available in a company’s database. At the heart of the model is a dynamic latent trait that can be interpreted as commitment, engagement, satisfaction, etc. So far we have examined the predictive validity of the model. However little has been said about the underlying process that drives both behaviors. While a comprehensive discussion of what the latent variable (as captured in a discretized form by the states of the HMM) actually represents, we can provide some evidence that is consistent with our contention that it capture the notion of commitment, etc.

In addition to priority booking privileges, membership of the organization’s Friends scheme gives the option to attend special events such as workshops, rehearsals, and educational events. Collecting information on the attendance of these events is not straightforward as they are organized by the Friends office and the data are not generally linked to the box-office data. We obtained information on event attendance for 2004 and extracted the records for those members belonging to the cohort analyzed in this paper. The rate of attendance of these events is very low compared to that of the general performances; on average, a member attends to 0.41 special events a year (0.10 per booking period), whereas the average attendance rate for general performances is 3.8.

On average, we would expect the attendance of such events to reflect a member’s “commitment”. Given that period 12 corresponds to the end of year 2004, we select the model predictions about state membership in period 12 (as shown in section 3.4) and compute the average attendance rate to special events by the members of each group. With reference to Table 12, we observe that the average number of special events attended is higher for higher commitment states.¹⁴ This average can be decomposed into i) the percentage of members attending at least one event, and ii) the average number of events attended *given* attendance of at least one event. We note that both of these quantities

¹⁴We do not consider the attendance rate for state 4 because the small size of the group.

increase with higher commitment states.

State	# customers	Average # events	Attending any event	Average given attendance
1	54	0.13	9%	1.4
2	135	0.44	23%	1.9
3	547	0.77	24%	3.2
4	2	–	–	–

Table 12: Attendance of other events

We recognize that this analysis has several limitations. First, we only consider one year and one booking period rather than performing a longitudinal analysis. If possible, one should get periodic information about attendance to special events, and match this information with the state membership estimates provided by the model. Second, the attendance of these events may not perfectly reflect commitment (or any other label we could assign to the latent trait that drives behavior in a particular setting). Nevertheless, we have shown that those customers identified by the model as more committed members attend special events more often than those who are assigned to lower states.

3.5 Generalizing the Model

There are certain aspects of this model that may appear to have been dictated by the specific empirical setting under study. We now briefly discuss the generalizability of the model.

First, there are situations where usage or consumption is not discrete. These are cases in which usage refers to time (e.g., minutes used in wireless contracts), expenditure (e.g., total amount spent), or other quantities (e.g., MB downloaded in an Internet data plan). The proposed model is easily applied in such settings, provided the distributional assumption of the usage process (derived in section 2.2) is modified. We would simply replace the Poisson or binomial with, say, a gamma or log-normal distribution.

Second, we may wish to incorporate time-invariant covariates (e.g., customer demographics) and/or time-varying covariates (e.g., marketing activities, competitors actions). In order to do so, we could simply make (3) or (5) a function of the covariates using,

say, an exponential link function. We could also use this information to explain observed heterogeneity in the transition behavior, as in Netzer et al. (2008).

4 Discussion

We have proposed a dynamic latent trait model in which usage and renewal behavior are modeled simultaneously by assuming that both behaviors are driven by the same (individual-level) underlying process that evolves over time. Dynamics in the underlying latent variable (which we label “commitment”) are captured using a hidden Markov model.

With regards to predicting churn behavior, the proposed model outperforms a set of benchmark models. It not only gives more accurate predictions in the short-term, but also forecasts churn further into the future. Given the usage component of the model, it is especially valuable in those contractual settings where future usage is not known in advance but is of importance, either because it directly affects revenue (e.g., credit cards, wireless contracts, arts organizations memberships), or because it affects service quality, which in turn affects customer retention and usage (e.g., gym memberships, DVD rental services).

We have quantified the importance of including dynamics in the latent variable and unobserved heterogeneity in usage behavior. We found that a model not considering heterogeneity in usage behavior increases the forecast error. Furthermore, we found that using a model that does not allow for dynamics in the latent variable yields a forecast error that is more than twice that associated with the proposed model.

Besides its methodological contribution, the model also provides important managerial insights. First, the retention and usage forecasts are vital inputs to any analysis of customer profitability. Second, the proposed model provides information that can be used by the marketer to target retention efforts individually and dynamically. Third, the model not only detects at-risk customers (potential churners) but also identifies highly committed customers. Fourth, because heterogeneity in usage behavior is allowed within each state, the model allows us to detect heavy users who might be at risk, and also light-users whose underlying level of commitment may be high. This information would be impossible to

extract from a segmentation scheme based entirely on observed transactional behavior. Moreover, model-derived insights can be updated easily as more behavior is observed, allowing the marketer to detect customers who recently changed their commitment to the organization. Finally, the model has been estimated using a freely available software, which facilitates its use by practitioners.

We recognize that this analysis is not free of limitations. First, we have not formally defined or measured the underlying construct that drives churn and usage and which makes it possible to model both processes simultaneously (even though we have called it commitment). Although the attempt of this work is to provide a methodological tool to predict churn and usage rather than identify such a construct, it would be very useful for the marketer to identify this underlying trait and also investigate *what* makes it change over time. To address this issue, customers' attitudes could be measured periodically and linked to the latent variable (using a factor-analytic measurement model). Incorporating this information might be costly. However, we hope that this research opens new venues to understand the dynamics between all customer's behaviors. Second, for the sake of model parsimony, we have proposed a homogeneous transition matrix. As such predictions about future commitment states might be misleading if the model is applied in a setting where there is a mix of "stable" and "volatile" customers. Nevertheless, the accuracy of the model forecasts provides support for our belief that the homogeneous HMM is appropriate for the empirical setting considered in this paper. A natural extension for the proposed method would be to incorporate unobserved individual heterogeneity in the (hidden) transition matrix. Moreover, one could even incorporate (when available) marketing actions carried out by the company that may affect the latent variable. The model could also be extended to accommodate seasonality in settings where such effects are present. Finally, another extension of our model is to include revenue as variable of interest. This could be done in two ways, either modeling revenue directly, as we have modeled usage (e.g., using a lognormal or gamma distribution) or modeling the link between usage and revenue.

Appendix A

The model is estimated in a Bayesian framework. We obtain estimates of all model parameters by drawing from the marginal posterior distributions, and use a data augmentation approach to deal with the latent states S_{it} .

Let $\boldsymbol{\Omega}$ denote all the model parameters, including the population parameters $(Q, \Pi, \boldsymbol{\theta}, r)$, the individual-level parameters $\boldsymbol{\alpha} = \{\alpha_i\}_{i=1, \dots, I}$, and the set of augmented paths of commitment states $\mathbf{s} = \{\tilde{s}_i\}_{i=1, \dots, I}$. The full joint posterior distribution can be written as:

$$f(\boldsymbol{\Omega}|\text{data}) \propto \left\{ \prod_{i=1}^I L_i^{\text{usage}}(\boldsymbol{\theta}, \alpha_i | S_i, \text{data}) \right\} f(\boldsymbol{\theta}) f(\boldsymbol{\alpha}|r) f(\mathbf{s}|Q, \Pi) f(r) f(Q) f(\Pi)$$

where $f(\boldsymbol{\theta})$ is the prior distribution of the commitment-state-specific usage parameters, $f(\boldsymbol{\alpha}|r)$ is the prior (or mixing) distribution for the α_i , which is assumed to follow a gamma distribution with shape and scale parameter r . The term $f(\mathbf{s}|Q, \Pi)$ refers to the distribution of the latent states, assumed to follow a hidden Markov process with renewal restrictions in periods $t = 1, n + 1, 2n + 1, \dots$

The terms $f(Q)$, $f(\Pi)$, and $f(r)$ are the (hyper)priors for the population parameters. Noninformative (vague) priors are used for all parameters. For the vector Q and all rows of the matrix Π we use Dirichlet prior with equal probabilities. We use a diffuse gamma distribution as a prior for the parameter r (shape and scale parameters being 0.01). In the Poisson specification we need to restrict the prior for $\boldsymbol{\theta}$ to ensure that $0 < \theta_1 < \theta_2 < \dots < \theta_K$. Therefore, we use a uniform prior over the interval (0,10) for θ_1 and reparameterize $\theta_\tau = \theta_{\tau-1} + e^{\gamma_\tau} \forall \tau > 1$. Diffuse normal priors (with mean 0 and variance 10000) are used for the γ_τ s. When estimating the binomial model, we also need to guaranty that all θ 's are lower than 1. To do so, we use a uniform (0,1) as prior for θ_1 , a uniform $(\theta_1, 1)$ for θ_2 , a uniform $(\theta_2, 1)$ for θ_3 and so on.

This is implemented using the freely available software WinBUGS (Lunn et al. 2000). We use the categorical distribution in WinBUGS to draw the augmented states, and control for the path restrictions (due to the contractual specification) by truncating the categorical distribution function in the renewal periods ($t = 1, n + 1, 2n + 1, \dots$). (Further

details of the estimation procedure and the associated WinBUGS code used to estimate both Poisson and binomial specifications are available from the authors on request.)

We burn-in the first 95,000 iterations and use the following 5,000 iterations to draw from the posterior marginal distributions. Convergence of the parameters was diagnosed visually, looking at the historical draws of all parameters, and also using the Gelman-Rubin statistic R , which compares the ratio of the pooled chain variance with the within chain variance, for a model estimated using multiple and disperse starting values.

Appendix B

Within both academic and practitioner circles, there is a tradition of building regression-type models for predicting churn and, to a lesser extent, usage (or related quantities). In this appendix, we describe the specification of the benchmark regression models used in our analyses.

As previously noted, the regressions model the behavior of interest as a function of the customer’s past behavior, frequently summarized in terms of their RFM characteristics. We operationalize these RFM characteristics in the following manner. *Recency* is defined as the number of periods since the last usage transaction (i.e., ticket purchase). *Frequency* is defined as the total number of usage transactions in the previous four periods. We also compute another measure of frequency, *Fsum*, which is the total number of transactions per customer over the entire period of interest. *Monetary value* is the average expenditure per transaction, where the average is computed over the previous four periods. We also compute *Msum*, the customer’s total spend over the entire period of interest. (In exploring possible model specifications, we also consider logarithmic transformations of these variables, as well as interactions between the RFM measures.)

Perhaps the most common approach to developing a churn model is to use a “cross-sectional” logistic regression with the last renewal observation as the dependent variable and RFM measures as covariates. In developing such a benchmark model, we selected the specification that provided the most accurate predictions. The associated parameter estimates are given in Table B1. We note that the recency variable is not a significant predictor by itself, although its interaction with frequency is a significant predictor of renewal behavior.

Given the nature of the usage data, we used a count model to develop our regression-based benchmark model. Given that we observe over-dispersion in our data, we used a negative binomial (NB) regression model, which can be viewed as a Poisson regression with a random effect that is distributed gamma with parameters (r, α) . We selected those individuals that were still members at the end of our calibration period. We used the number of transactions in the last period (11) as the dependent variable and the RFM

	Coef.	Std. Err.
Intercept	0.746	0.294
Recency	0.016	0.076
Fsum	0.058	0.017
Msum	0.002	0.000
Recency \times Fsum	-0.071	0.015
Frequency \times Monetary value	-0.002	0.000
LL	-327.4	

Table B1: Cross-sectional logistic regression

measures as predictors. The parameter estimates are presented in Table B2. At first glance, it might look surprising that the coefficients of both frequency and monetary value are negative. However, these parameters are not the true effect of past frequency and monetary value on current usage because the interactions with recency are very strong. Taking this interaction into account, the effect of both frequency and monetary value is strictly positive. We observe a high degree of unobserved heterogeneity in usage behavior; the gamma parameters $r = 1.94$ and $\alpha = 0.68$ map to a random effect with a mean of 2.87 and a variance of 4.25.

	Coef.	Std. Err.
Random effect		
r	1.941	0.373
α	0.675	0.172
Recency	-0.740	0.073
Frequency	-0.035	0.019
Monetary value	-0.009	0.001
Recency \times Frequency	0.066	0.017
Recency \times Monetary	0.003	0.000
Frequency \times Monetary	0.001	0.000
LL	-783.5	

Table B2: Cross-sectional NB regression

Noting that the longitudinal nature of our dataset gives us several observations per individual, and not just the information for the most recent period, we can extend the cross-sectional models and estimate longitudinal models using (when available) more than one observation per customer.

Using these panel data, we estimate a logistic regression using observed renewal behavior for all the periods, not just the most recent one; this gives us several observations for those customers that have renewed at least once. We allow for unobserved heterogeneity in renewal behavior using a normal random effect. Table B3 shows the parameter estimates for the (longitudinal) random effect churn model. The sign and magnitude of all covariates are consistent with the results obtained in the cross-sectional specification. (Note that variance of the random effect is not significant.)

	Coef.	Std. Err.
Random effect		
μ	-0.084	0.176
σ^2	0.000	0.152
Recency	0.091	0.049
Frequency	0.059	0.030
Msum	0.003	0.000
Recency \times Frequency	-0.099	0.027
Recency \times Monetary	-0.001	0.000
Frequency \times Monetary	-0.003	0.000
LL	-775.6	

Table B3: Panel logistic regression

Similarly, we estimate a NB regression using transaction behavior from all preceding periods—see Table B4. The results are consistent with those obtained in the cross-sectional model.

	Coef.	Std. Err.
Random effect		
r	1.142	0.098
α	0.716	0.067
Recency	-0.352	0.031
Frequency	-0.030	0.005
Monetary value	-0.004	0.001
Recency \times Frequency	0.040	0.005
Recency \times Monetary	0.002	0.000
Frequency \times Monetary	0.000	0.000
LL	-6725.5	

Table B4: Panel NB regression

References

- Ascarza, E., A. Lambrecht, N. Vilcassim. 2009. When talk is “free”: An analysis of subscriber behavior under two- and three-part tariffs. Working Paper, London Business School.
- Athanassopoulos, A.D. 2000. Customer satisfaction cues to support market segmentation and explain switching behavior. *J. Business Res.* **47**(3) 191–207.
- Berry, M.J.A., G.S. Linoff. 2004. *Data Mining Techniques: For Marketing, Sales, and Customer Relationship Management*. John Wiley & Sons, Indianapolis, IN.
- Bhattacharya, C.B. 1998. When customers are members: Customer retention in paid membership context. *J. Marketing* **26**(Winter) 31–44.
- Blattberg, R.C., G. Getz, J.S. Thomas. 2001. *Customer Equity: Building and Managing Relationships as Valuable Assets*. Harvard Business School Press, Boston, MA.
- Blattberg, R.C., B-D. Kim, S.A. Neslin. 2008. *Database Marketing. Analyzing and Managing Customers*. Springer, New York, NY.
- Bolton, R.N. 1998. A dynamic model of duration of the customer’s relationship with a continuous service provider: The role of satisfaction. *Marketing Sci.* **17**(1) 45–65.
- Bolton, R.N., P.K. Kannan, M.D. Bramlett. 2000. Implications of loyalty program membership and service experiences for customer retention and value. *J. Academy Marketing Sci.* **28**(Winter) 95–108.
- Bolton, R.N., K.N. Lemon. 1999. A dynamic model of customers’ usage of services: Usage as a n antecedent and consequence of satisfaction. *J. Marketing Res.* **36**(2) 171–186.
- Bolton, R.N., K.N. Lemon, P.C. Verhoef. 2004. The theoretical underpinnings of customer asset management: A framework and propositions for future research. *J. Academy Marketing Sci.* **32**(3) 271–292.
- Brannas, K., P. Johansson. 1996. Panel data regression for counts. *Statistical Papers* **37** 191–213.
- Chintagunta, P.K. 1993. Investigating purchase incidence, brand choice and purchase quantity decisions of households. *Marketing Sci.* **12**(Spring) 184–204.
- Congdon, P. 2003. *Bayesian Statistical Modelling*. John Wiley & Sons, Chichester, England.
- Diggle, P., M.G. Kenward. 1994. Informative drop-out in longitudinal data analysis. *App. Stat.* **43**(1) 49–93.
- Fader, P.S., B.G.S. Hardie, C-Y. Huang. 2004, A dynamic changepoint model for new product sales forecasting. *Marketing Sci.* **23**(Winter) 50–65.
- Fader, P.S., B.G.S. Hardie, K.L. Lee. 2005. RFM and CLV: Using iso-value curves for customer base analysis. *J. Marketing Res.* **42**(November) 415–430.

- Garbarino, E., M.S. Johnson. 1999. The different roles of satisfaction, trust, and commitment in customer relationships. *J. Marketing* **63**(April) 70–87.
- Gourieroux, C., J. Jasiak. 2001. Dynamic factor models. *Econometric Reviews* **20**(4) 385–424.
- Gruen, T.W., J.O. Summers, F. Acito. 2000. Relationship marketing activities, commitment, and membership behaviors in professional associations. *J. Marketing* **64**(July) 34–49.
- Hanemann, W.M. 1984. Discrete/continuous models of consumer demand. *Econometrica* **52**(3) 541–561.
- Hashemi, R., H. Janqmin-Dagga, D. Commenges. 2003. A latent process model for joint modeling of events and marker. *Lifetime Data Analysis* **9**(December) 331–343.
- Hausman, J., B.H. Hall, Z. Griliches. 1984. Econometric models for count data with an application to the patents-R&D relationship. *Econometrica* **52**(4) 909–938.
- Henderson, R., P. Diggle, A. Dobson. 2000. Joint modelling of longitudinal measurements and event time data. *Biostatistics* **1**(4) 465–480.
- Kim, H.S., C.H. Yoon. 2004. Determinants of subscriber churn and customer loyalty in the Korean mobile telephony market. *Telecommunications Policy* **28**(40095) 751–765.
- Krishnamurthi, L., S.P. Raj. 1988. A model of brand choice and purchase quantity price sensitivities. *Marketing Sci.* **7**(Winter) 1–20.
- Lachaab, M., A. Ansari, K. Jedidi, A. Trabelsi. 2006. Modeling preference evolution in discrete choice models: A Bayesian state-space approach. *Quant. Marketing and Econ.* **4** 57–81.
- Larivière, B., D. Van den Poel. 2005. Predicting customer retention and profitability by using random forests and regression forests techniques. *Expert Systems with Applications* **29**(2) 472–484.
- Lemon, K.N., T.B. White, R.S. Winer. 2002. Dynamic customer relationship management: Incorporating future considerations into the service retention decision. *J. Marketing* **66**(January) 1–14.
- Liechty, J.C., D.K.H. Fong, W.S. DeSarbo. 2005. Dynamic models incorporating individual heterogeneity: Utility evolution in conjoint analysis. *Marketing Sci.* **24**(Spring) 285–293.
- Liu, L., X. Huang. 2009. Joint analysis of correlated repeated measures and recurrent events processes in the presence of death, with application to a study on acquired immune deficiency syndrome. *App. Stat.* **58**(1) 65–81.
- Lu, J. 2002. Predicting customer churn in the telecommunications industry—An application of survival analysis modeling using SAS®. *SAS User Group Internat. (SUGI27) Online Proceedings* 114–27.

- Lunn, D.J., A. Thomas, N. Best, D. Spiegelhalter. 2000. WinBUGS—A Bayesian modelling framework: Concepts, structure, and extensibility. *Stat. and Computing* **10**(4) 325–337.
- Moe, W.W., P.S. Fader. 2004a. Capturing evolving visit behavior in clickstream data. *J. Interactive Marketing* **18**(Winter) 5–19.
- Moe, W.W., P.S. Fader. 2004b. Dynamic conversion behavior at e-commerce sites. *Management Sci.* **50**(March) 326–335.
- Montgomery, A.L., S. Li, K. Srinivasan, J.C. Liechty. 2004. Predicting online purchase conversion using web path analysis. *Marketing Sci.* **23**(4) 579–595.
- Moon, S., W.A. Kamakura, J. Ledolter. 2007. Estimating promotion response when competitive promotions are unobservable. *J. Marketing Res.* **44**(3) 503–515.
- Morgan, R.Y.H., S. Hunt. 1994. The commitment-trust theory of relationship marketing. *J. Marketing* **58**(July) 20–38.
- Mozer, M.C., R. Wolniewicz, D.B. Grimes, E. Johnson, H. Kaushansky. 2000. Predicting subscriber dissatisfaction and improving retention in the wireless telecommunications industry. *IEEE Trans. Neural Networks* **11**(3) 690–696.
- Nam, S., P. Manchanda, P.K. Chintagunta. 2007. The effects of service quality and word of mouth on customer acquisition, retention and usage. (<http://ssrn.com/abstract=969770>).
- Narayanan, S., P.K. Chintagunta, E.J. Miravete. 2007. The role of self selection and usage uncertainty in the demand for local telephone service. *Quantitative Marketing and Econom.* **5**(1) 1–34.
- Netzer, O., J.M. Lattin, V. Srinivasan. 2008. A hidden Markov model of customer relationship dynamics. *Marketing Sci.* **27**(March-April) 185–204.
- Newton, M.A., A.E. Raftery. 1994. Approximate Bayesian inference with the weighted likelihood bootstrap. *J. Roy. Stat. Soc. B* **56**(1) 3–48.
- Parr Rud, O. 2001. *Data Mining Cookbook*. John Wiley & Sons, New York, NY.
- Poulsen, C.S. 1990. Mixed Markov and latent Markov modelling applied to brand choice behaviour. *Internat. J. Res. in Marketing* **7** 5–19.
- Reinartz, W., J.S. Thomas, V. Kumar. 2005. Balancing acquisition and retention resources to maximize customer profitability. *J. Marketing* **69**(January) 63–79.
- Rust, R.T., V.A. Zeithaml, K.N. Lemon. 2001. *Driving Customer Equity: How Customer Lifetime Value Is Reshaping Corporate Strategy*. The Free Press, New York, NY.
- Sabavala, D.J., D.G. Morrison. 1981. A nonstationary model of binary choice applied to media exposure. *Management Sci.* **27**(June) 637–657.
- Schweidel, D.A., P.S. Fader, E.T. Bradlow. 2008. Understanding service retention within and across cohorts using limited information. *J. Marketing* **72**(1) 82–94.

- Smith, A., P.A. Naik, C.L. Tsai. 2006. Markov-switching model selection using Kullback–Leibler divergence. *J. Econometrics* **134**(2) 553–577.
- Van Heerde, H.J., C.F. Mela, P. Manchanda. 2004. The dynamic effect of innovation on market structure. *J. Marketing Res.* **41**(May) 166–183.
- Verhoef, P.C. 2003. Understanding the effect of customer relationship management efforts on customer retention and customer share development. *J. Marketing* **67**(October) 30–45.
- West, M., J. Harrison. 1997. *Bayesian Forecasting and Dynamic Models*, 2nd ed. Springer, New York, NY.
- Wübben, M., F. Wangenheim. 2008. Instant customer base analysis: Managerial heuristics often get it right. *J. Marketing* **72**(3) 82–93.
- Xu, J., S.L. Zeger. 2001. Joint analysis of longitudinal data comprising repeated measures and times to events. *App. Stat.* **50**(3) 375–387.