PROBLEM SET 7 (WILL NOT BE GRADED GIVEN THE LATE POSTING)

(All Exercises are references to the November 18, 2017 version of *Foundations of Algebraic Geometry* by R. Vakil.)

Problem 1. Suppose that k is an algebraically closed field of characteristic 0, and suppose that C is a connected smooth curve over k admitting an unramified morphism $\pi : C \to \mathbb{A}^1_k$. Prove that π is an open embedding. (See Exercise 21.7.F for a little discussion of this. Once you've done this problem, you might be interested in thinking about what happens if you delete a (closed) point from $\mathbb{A}^1_k$ and try again to understand unramified covers.)

Problem 2. Suppose that k is an algebraically closed field of characteristic not equal to 3. Let $g \geq 0$. Suppose that $x_1, \ldots, x_{g+2}$ are distinct closed points in $\mathbb{P}^1_k$. Count the number of isomorphism classes of degree 3 maps of irreducible smooth projective curves $\pi : C \to \mathbb{P}^1_k$ that are branched precisely over the x_i and such that the extension of function fields is Galois. (The answer should end up being $(2^{g+1} - (-1)^{g+1})/3$.)

Problem 3. Suppose that k is an algebraically closed field of characteristic 0, and suppose that $C \subset \mathbb{P}^2_k$ is a smooth plane curve of degree d. Let $p \in \mathbb{P}^2_k$ be a closed point. Count the number of tangent lines to C that pass through a "general" such p; in other words, your answer should be true on some open dense subset of $\mathbb{P}^2_k$. (The answer should end up being $d(d-1)$ - you can obtain this either by using the formula for the genus of C along with the Riemann-Hurwitz formula on the projection of C from p, or by directly interpreting these tangent lines as coming from the intersection of a degree $d-1$ curve with C. In the latter case, this computation along with the Riemann-Hurwitz formula gives yet another computation of the genus of a smooth plane curve.)