PROBLEM SET 2 (DUE ON THURSDAY, MARCH 7)

(All Exercises are references to the November 18, 2017 version of *Foundations of Algebraic Geometry* by R. Vakil.)

Problem 1. Exercise 16.3.H (the projection formula)

Problem 2. Exercise 16.4.J (dimensions of images of maps from projective space)

Problem 3. Exercise 16.5.B (extending over regular codimension 1 sets - if you want, instead of writing down a full proof you can just explain how to modify the proof of Theorem 16.5.1)

Problem 4. Exercise 16.6.C (very ample $\otimes$ base-point-free is very ample)

Problem 5. Let $S_\bullet = \mathbb{C}[x, y, z, w]/(xw - yz)$ be a graded ring, where we take $\deg x = \deg y = 0$ and $\deg z = \deg w = 1$. Let $M_\bullet$ be the graded $S_\bullet$-module given by the ideal (xw) of $S_\bullet$. Let $\widetilde{M_\bullet}$ be the corresponding quasicoherent sheaf on $\operatorname{Proj} S_\bullet$ (as defined in Section 15.1). Show that $\widetilde{M_\bullet}$ is a line bundle and compute its base locus. (Hint: Use the affine charts $\operatorname{Proj} S_\bullet = D(z) \cup D(w)$.) (Addendum: If you prefer, you can view $M_\bullet$ as the degree shift $S(-1)_\bullet$, so $\widetilde{M_\bullet}$ also goes by the name $\mathcal{O}_{\operatorname{Proj} S_\bullet}(-1)$.)