

INFO ON FINAL PAPER FOR 18.726

First, the basics: undergraduates taking 18.726 for credit this semester are expected (1/4 of final grade) to write a short (3-10 pages) final paper. The paper will be due on the last day of class, May 16.

Algebraic geometry is a very wide subject - there are lots of different directions to go, and many things that we won't have time to do this semester. I see this paper as an opportunity for you to pick one of these directions that interests you and learn a little more about it. You should pick a topic, learn about it, and then pick a key result or interesting example and write an exposition of the parts of the topic needed to get to the result/example (assuming as base knowledge just what we have learned in class).

It certainly isn't required to write down every single necessary detail along the way - you will have to use your judgment for that, just as any writer of math does. The idea is just to write a treatment of the topic that feels clear and natural to you. In cases where the topic corresponds to sections in Vakil, this will probably involve figuring out how to prove some of the exercises (or figuring out alternative ways of getting to the target results).

Here are some sample options for topics (though you are welcome to choose your own topic - if there's some specific direction in which you'd like to go, please let me know and I can help you select a topic):

- basic intersection theory (Chapter 20 of Vakil, possibly with some of the results in Section 20.4 as a goal)
- blowups/singularities (Chapter 22 of Vakil and also possibly some of Chapter 29; one possibility is to work out some of the ADE-surface singularity examples in 22.4.F and 22.4.5)
- derived functors (Chapter 23 of Vakil - one possible goal is Theorem 23.5.1)
- cubic surfaces (Chapter 27 of Vakil - the idea would be to understand and sketch the argument behind either Theorem 27.1.1 or Theorem 27.1.2)
- the classification of genus 5 curves as hyperelliptic, trigonal, or complete intersections of three quadrics (one possible source for this is Geometry of Algebraic Curves by Arbarello, Cornalba, Griffiths, and Harris, which outlines how to prove a more general theorem (the Enriques-Babbage Theorem) in Section III.3)
- Hilbert schemes (there is a little bit about these in Section 28.3 of Vakil, but you will need to look elsewhere for an actual construction - one possible reference is <https://homepages.warwick.ac.uk/staff/D.Maclagan/papers/HilbertSchemesNotes.pdf>)
- connections with topology, e.g. genus of an algebraic curve equals topological genus of the corresponding Riemann surface (Exercise 21.7.I outlines how you might prove this single fact, but you might want to do more than just this - for example, can you give a (probably non-rigorous) explanation of why compact Riemann surfaces should all come from algebraic curves? I can give a reference or hints for this if desired.)
- low genus examples of moduli of stable curves (this is a very combinatorial and beautiful subject - I like Vakil's introduction to this in the first 8 pages or so of <http://math.stanford.edu/~vakil/files/notices/ams4.pdf>, but there are many other

references out there. One possible topic is describing how $\overline{M}_{0,n+3}$ can be viewed as an iterated blowup of \mathbb{P}^n , while another is describing the relationship between \overline{M}_2 and $\overline{M}_{0,6}$ - I can provide more details on request.)

I'm also happy to talk about any of these topics that might interest you even if you aren't actually writing a paper about them!