

PROBLEM SET 5 (DUE ON THURSDAY, NOV 1)

(All Exercises are references to the November 18, 2017 version of *Foundations of Algebraic Geometry* by R. Vakil.)

- Problem 1.** Exercise 6.3.K (morphisms from Spec of a local ring)
- Problem 2.** Exercise 6.3.M (some morphisms to \mathbb{P}^n)
- Problem 3.** Describe all morphisms of k -schemes $\pi : \mathbb{P}_k^1 \rightarrow \mathbb{P}_k^1$. (Hint: if π maps the generic point to the generic point, consider the induced map of stalks there.)
- Problem 4.** Let $\phi : \mathbb{C}[y_0, y_1, y_2] \rightarrow \mathbb{C}[x_0, x_1, x_2]$ be the map of graded rings (where each x_i or y_i has degree 1) given by $(\phi(y_0), \phi(y_1), \phi(y_2)) = (x_1x_2, x_0x_2, x_0x_1)$. Let $\phi^* : U \rightarrow \mathbb{P}_{\mathbb{C}}^2$ be the map on Proj induced by ϕ , where $U = \mathbb{P}_{\mathbb{C}}^2 \setminus V(x_1x_2, x_0x_2, x_0x_1)$ is an open subscheme of $\mathbb{P}_{\mathbb{C}}^2$. Describe the set $V \subseteq \mathbb{P}_{\mathbb{C}}^2$ of all points $p \in \mathbb{P}_{\mathbb{C}}^2$ such that there is exactly one $q \in U$ with $\phi^*(q) = p$.
- Problem 5.** Let k be a field. Let $X = \text{Spec } \mathbb{Z}[x, y]/xy$. Define a natural map $X(k[\epsilon]/\epsilon^2) \rightarrow X(k)$, where $X(A)$ is the set of A -valued points of X , and describe the fibers of this map.