PROBLEM SET 1 (DUE ON THURSDAY, JANUARY 19)

(All Exercises are references to the December 31, 2022 version of *Foundations of Algebraic Geometry* by R. Vakil.)

Problem 1. Let X be a reduced scheme and let $\mathcal{F}$ be a finite type quasicoherent sheaf on X with rank r at every point of X. Prove that $\mathcal{F}$ is a vector bundle of rank r. Give a counterexample if X is not assumed to be reduced. (Hint: use Geometric Nakayama's Lemma to find open affines such that $\mathcal{F}(\operatorname{Spec} A)$ is generated by r sections, then use the reduced assumption to show these sections can't have any relations.)

Problem 2. Exercise 14.3.F (being a vector bundle can be checked on stalks - note here $\mathcal{F}$ is assumed to be finitely presented, i.e. the modules $\mathcal{F}(\operatorname{Spec} A)$ are finitely generated and finitely presented - this is an intermediate condition between being finitely generated and being coherent)

Problem 3. Exercise 14.3.L (degrees of finite morphisms at points)