PROBLEM SET 8 (DUE ON THURSDAY, NOV 2)

(All Exercises are references to the July 31, 2023 version of *Foundations of Algebraic Geometry* by R. Vakil.)

Problem 1. (a) Construct a morphism

$$\pi_1 : \mathbb{A}_k^{n+1} \setminus \{(0, \ldots, 0)\} \to \mathbb{P}_k^n$$

that induces the classical quotient map $k^{n+1} \setminus \{0\} \to \mathbb{P}_k^n(k)$ on k-valued points. Is π_1 affine?

(b) Extend π_1 to a morphism

$$\pi_2 : \mathbb{P}_k^{n+1} \setminus \{[1 : 0 : \cdots : 0]\} \to \mathbb{P}_k^n.$$

Is π_2 affine?

(c) Set $k = \mathbb{C}$ and $n = 1$. Let $X = V(x_0x_1 - x_2^2) \subseteq \mathbb{P}_{\mathbb{C}}^2$, a projective hypersurface of degree 2 passing through $[1 : 0 : \cdots : 0]$. Restrict π_2 to a morphism

$$\pi_3 : X \setminus \{[1 : 0 : \cdots : 0]\} \to \mathbb{P}_{\mathbb{C}}^1.$$

Show that π_3 is an open embedding.

(d) Extend π_3 to an isomorphism

$$\pi_4 : X \to \mathbb{P}_{\mathbb{C}}^1.$$

Problem 2. Exercise 9.1.I(d) (an example of scheme-theoretic intersection not distributing over scheme-theoretic union)

Problem 3. A *quadric* in $\mathbb{A}_k^n$ is a closed subscheme $V(f)$ cut out by a single polynomial of degree two. Give an example of two quadrics in $\mathbb{A}_{\mathbb{C}}^2$ intersecting in a single point (and nowhere else), and compute the scheme-theoretic intersection. Then give a second example of this, with scheme-theoretic intersection not isomorphic (as schemes) to that in your first example.