

PROBLEM SET 5 (DUE ON THURSDAY, OCT 12)

(All Exercises are references to the July 31, 2023 version of *Foundations of Algebraic Geometry* by R. Vakil.)

- Problem 1.** Exercise 5.1.B (irreducible closed subsets of general schemes are closures of points)
- Problem 2.** Let k be a field. Let $X = \text{Spec } \mathbb{Z}[x, y]/xy$. Define a natural map $X(k[\epsilon]/\epsilon^2) \rightarrow X(k)$, where $X(A)$ is the set of A -valued points of X (i.e. morphisms from $\text{Spec } A$ to X). Compute the fibers of this map - you should find that for any $p \in X(k)$, the points in the fiber above p look like a k -vector space. (This is a preview of the notion of the *tangent space* to a scheme.)
- Problem 3.** Exercise 8.1.D (fiber products of open embeddings - there is a discussion of fiber products in Section 1.3.6)
- Problem 4.** Exercise 8.3.C(a) (quasicompactness is affine-local on the target)