

## PROBLEM SET 8 (DUE ON THURSDAY, NOV 5)

(All Exercises are references to the November 18, 2017 version of *Foundations of Algebraic Geometry* by R. Vakil.)

- Problem 1.** Exercise 4.5.E(a) (prime ideals of  $(S_{\bullet}[\frac{1}{f}])_0$ )
- Problem 2.** Exercise 8.2.C (closed embeddings of projective schemes)
- Problem 3.** Is  $\text{Proj } k[x, y]/(x^2y)$  affine, where  $x$  and  $y$  have degree 1? Is it reduced?
- Problem 4.** A *quadric* in  $\mathbb{P}_k^n$  is a closed subscheme cut out by a single homogeneous polynomial of degree two (see 8.2.2). Give an example of two quadrics in  $\mathbb{P}_{\mathbb{R}}^2$  intersecting in a single point, and compute the scheme-theoretic intersection. Then give a second example of this, with scheme-theoretic intersection not isomorphic (as schemes) to that in your first example. Then give a third example with intersection not isomorphic to either of the first two! (Changes from last week's problem:  $\mathbb{P}^2$  instead of  $\mathbb{A}^2$ ;  $\mathbb{R}$  instead of  $\mathbb{C}$ ; three examples instead of two examples. It may be helpful to note that Bezout's theorem now applies and says that the intersection must be of the form  $\text{Spec } A$ , where  $A$  is an  $\mathbb{R}$ -algebra that is 4-dimensional as an  $\mathbb{R}$ -vector space and has exactly one prime ideal.)