

PROBLEM SET 4 (DUE ON THURSDAY, OCT 8)

(All Exercises are references to the November 18, 2017 version of *Foundations of Algebraic Geometry* by R. Vakil.)

Problem 1. Exercise 4.3.G (functions on locally ringed spaces)

Problem 2. Let $X_1 = \text{Spec } k[x, y]$ and $X_2 = \text{Spec } k[w, z]$ be two copies of the affine plane over a field k . Let X be the scheme formed by gluing X_1 and X_2 along the isomorphism of open subschemes $\text{Spec } k[x, x^{-1}, y] \cong \text{Spec } k[w, w^{-1}, z]$ induced by the ring isomorphism $k[x, x^{-1}, y] \cong k[w, w^{-1}, z]$ given by $x \mapsto w, y \mapsto w^{-1}z$. Compute the ring of global sections of the structure sheaf of X . Is X affine?

Problem 3. Exercise 6.3.K (morphisms from Spec of a local ring)

Problem 4. Describe all morphisms of $\pi : \mathbb{P}_k^1 \rightarrow \mathbb{P}_k^1$ that commute with the natural morphism $\mathbb{P}_k^1 \rightarrow \text{Spec } k$. (Note: this is saying that π is a “morphism of k -schemes”.) Hint: if π maps the generic point to the generic point, consider the induced map of stalks there.)