Welcome! (Math 631)

- plan for today:
  - go over the course structure
  - classical vs modern alg. geom, explain progression for first few weeks.

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2 websites:

Canvas site: - Zoom info/links/recordings
- announcements

main website: everything else
- link to textbook ("Foundations of Alg. Geom." by Ravi Vakil)

(links to other references)

- syllabus
- problem sets

Problem sets: weekly starting next week
- no exams

Office hours: - on Zoom
This week's schedule: 2:30-3:30 TWR
- survey after class about which times are bad/good.
- can always e-mail to schedule another meeting.

Prerequisites:
- topology: (open covers, bases ("small open n'hoods"))
- ring theory: (prime ideals, localizations tensor product)
  - trying not to need much commutative alg.
    - will use a few results as black boxes
- category theory (language)
  - survey asks how much category theory you are familiar with.

Chapter 1 of Vakil

- introduction to category theory
- recommend reading over this a little
  (with exception of the final starred section on spectral sequences, which we will not need)

Rough prediction for what we'll cover this semester: Chapters 2 - ~14 of Vakil

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Classical algebraic geometry:

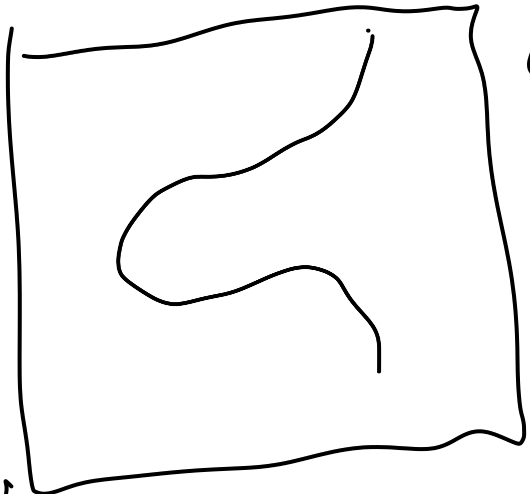
variables $x_1, \dots, x_n$

polynomials $f_1, \dots, f_m \in \mathbb{C}[x_1, \dots, x_n]$

$\rightsquigarrow$ study

$$\{(x_1, \dots, x_n) \in \mathbb{C}^n \mid f_1 = \dots = f_m = 0\}$$

"algebraic set" "affine variety"



$\mathbb{C}^2$

$y^2 - x^3 - x = 0$

study geometry/topology of this subset of $\mathbb{C}^n$

modern viewpoint (schemes): same picture/intuition, but don't just think of it as an analytic subset of $\mathbb{C}^n$

something called

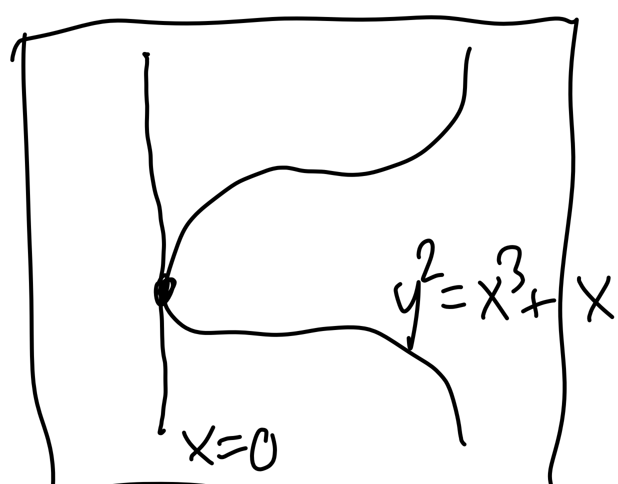
$$\underbrace{\mathrm{Spec}}_{\text{functor creating an "affine scheme"}} \ \underbrace{\mathbb{C}[x,y]/(y^2-x^3-x)}_{\text{ring}} = \text{top. space with additional structure (a "sheaf" of rings)}$$

advantages of schemes (over alg. sets/variety)

- cleaner way to describe blend of alg. and geom. structures.
- works well with arbitrary base fields (or even rings), not just $\mathbb{C}$.

  ($x^2+y^2=0$ and $x=y=0$ have the same solutions in $\mathbb{R}^2$, but $\mathbb{R}[x,y]/(x^2+y^2)$ and $\mathbb{R}[x,y]/(x,y)$ are very different as rings)

1) much easier to glue schemes together than to glue algebraic sets together.

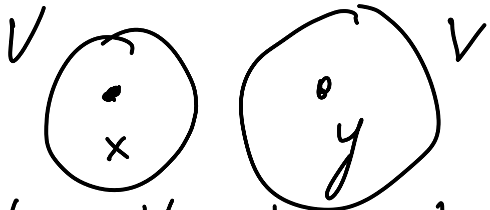
2) 

Like to say that these curves are tangent at $(0,0)$

Schemes: intersection is "point with additional structure" disadvantages:

- definition takes some preliminary work $\boxed{\text{sheaves}}$
- topological spaces appearing are $\underbrace{\text{unusual}}_{\text{not Hausdorff}}$

(recall: X is Hausdorff if any two points "can be separated")



It will take a bit to build up intuition for non-Hausdorff spaces.

Progression at start:

- start with developing the basic theory of <u>sheaves</u>

A <u>sheaf</u> is a topological concept

- type of structure associated to open subsets of a top. space that behaves well under gluing.

- <u>Example:</u>

X = top. space

$\mathcal{F}$ = sheaf on X:

for any open set $U \subset X$, there is a set

$\mathcal{F}(U)$

example: $\mathcal{F}(U) = \{\text{continuous functions } f: U \to \mathbb{R}\}$

another example: X = smooth manifold,

$\mathcal{F}(U) = \{\text{smooth functions } f: U \to \mathbb{R}\}$

Scheme = top. space + sheaf of rings
+ additional condition involving localizations of rings.

will also need to understand morphisms of schemes.

After class: post link to a survey as an announcement on Canvas.

Please fill out the survey today or tomorrow