Magnetic Tower Model for Long Gamma-Ray Bursts

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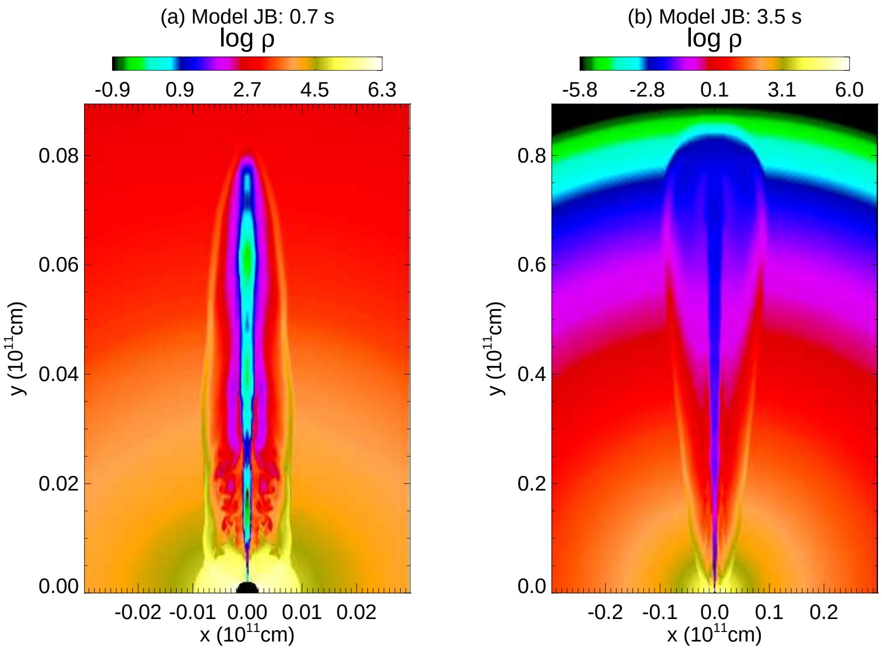
Collapsar Model for GRBs

(Woosley 1993; Paczynski 1998; MacFadyen & Woosley 1999)



Zhang, Woosley, MacFadyen (2003)

- Core of a massive star collapses into black hole.
- Rotating stellar material falls towards the black hole and forms an accretion disk, $\dot{M} \sim 0.1 M_{\rm Sun}/{\rm sec}$!
- Accretion disk cools by emitting neutrinos. Neutrinos annihilate above the disk and form a relativistically-hot $e^+ - e^-$ fireball.
- The fireball drives a hydrodynamic jet through the star.
- High-pressure cocoon forms behind the shock and keeps the jet collimated.

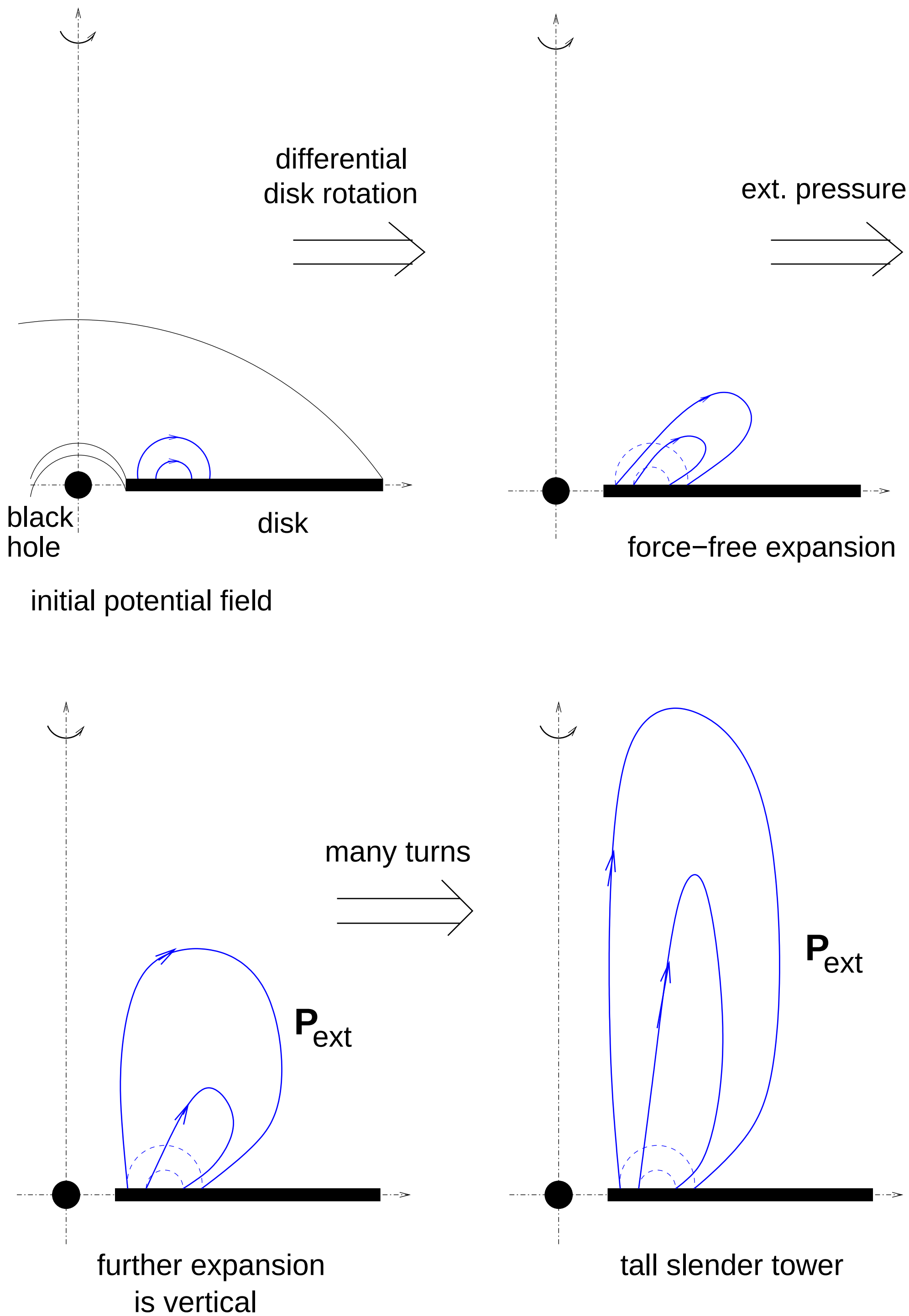
Role of Magnetic Fields in Collapsars

- Strong magnetic fields ($B \sim 10^{15}$ Gauss) unavoidably generated during explosions of rotating massive stars (long-duration GRBs and core-collapse SNe)
  e.g., van Putten & Levinson (2003); Akiyama et al. (2003); Wheeler et al. (2005)

- These strong magnetic fields change explosion dynamics and also affect neutrino transport
  e.g., Paczynski 1998; MacFadyen & Woosley (1999); van Putten & Levinson (2003); Akiyama et al. (2003); Wheeler et al. (2005); Proga et al. (2003); Sawai et al. (2005); Ardeljan et al. (2005)

- There are observational and theoretical reasons to believe that GRB outflows are magnetically-dominated
  e.g., Lyutikov & Blandford (2003); Lyutikov (2004); Giannios & Spruit (2004); Spruit & Drenkhahn (2004)

Magnetic Tower Model: a Sketch
(*Lyndon-Bell 1996*)



Magnetic Tower Model: Scalings

input parameters: Ψ, $\Delta\Omega$, $P_{\rm ext}$ $\Rightarrow$

tower parameters: B_0, R_0, $V_{\rm top}$

- **magnetic flux conservation:**

$$B_0 \sim \frac{\Psi}{R_0^2}$$

- **horizontal pressure balance:**

$$\frac{B_0^2}{8\pi} \sim P_{\rm ext}$$

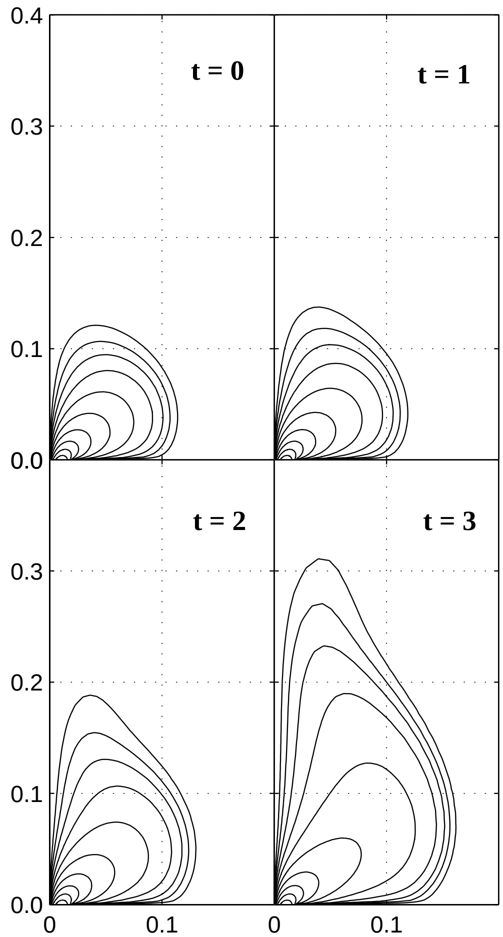
- **differential rotation:**

$$N = \Delta\Omega t/2\pi = \frac{\Psi_{\rm tor}}{\Psi_{\rm pol}} \quad \Rightarrow \quad V_{\rm top} \sim \Delta\Omega R_0$$

Magnetic Towers in Simulations

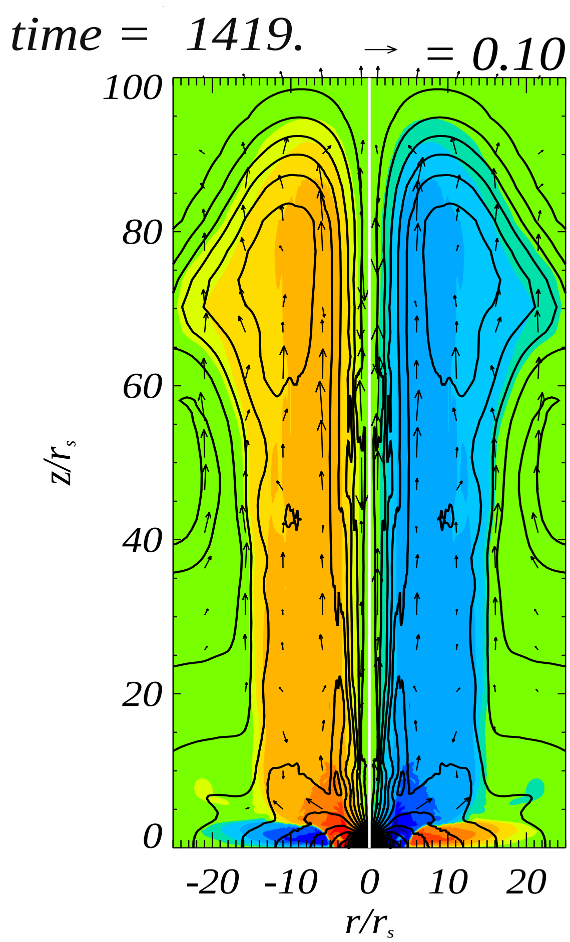
Sequence of Magnetostatic Equilibria:

Li, Lovelace, Finn, Colgate (2001)



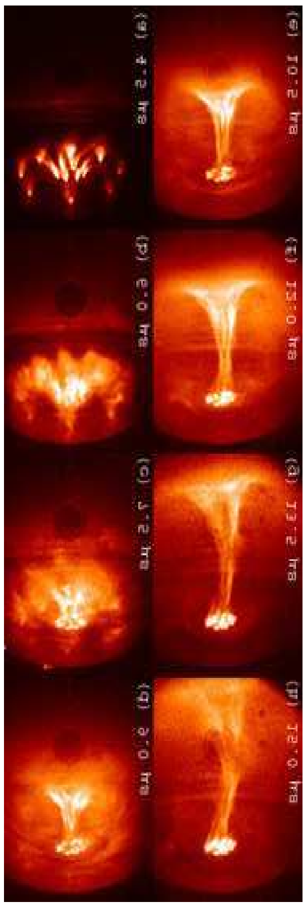
Axisymmetric MHD Simulations:

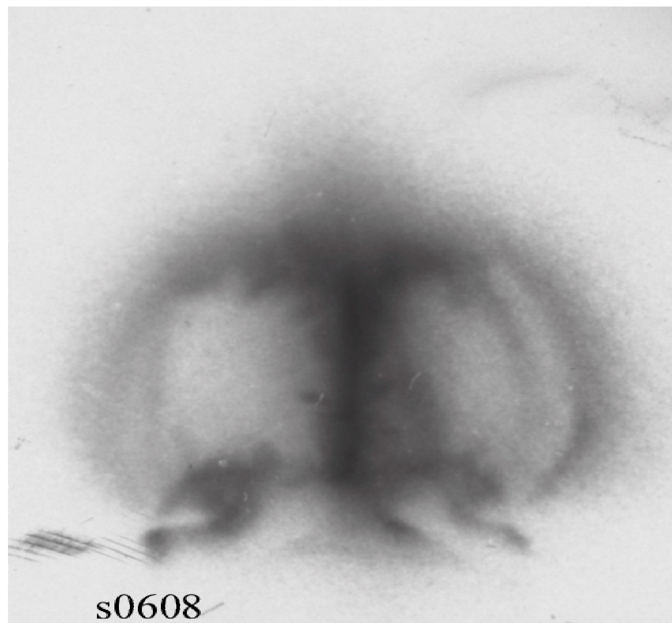
Kato, Hayashi, Matsumoto (2004)



Magnetic Towers in the Lab

Hsu & Bellan (2002) *Lebedev et al. (2005)*



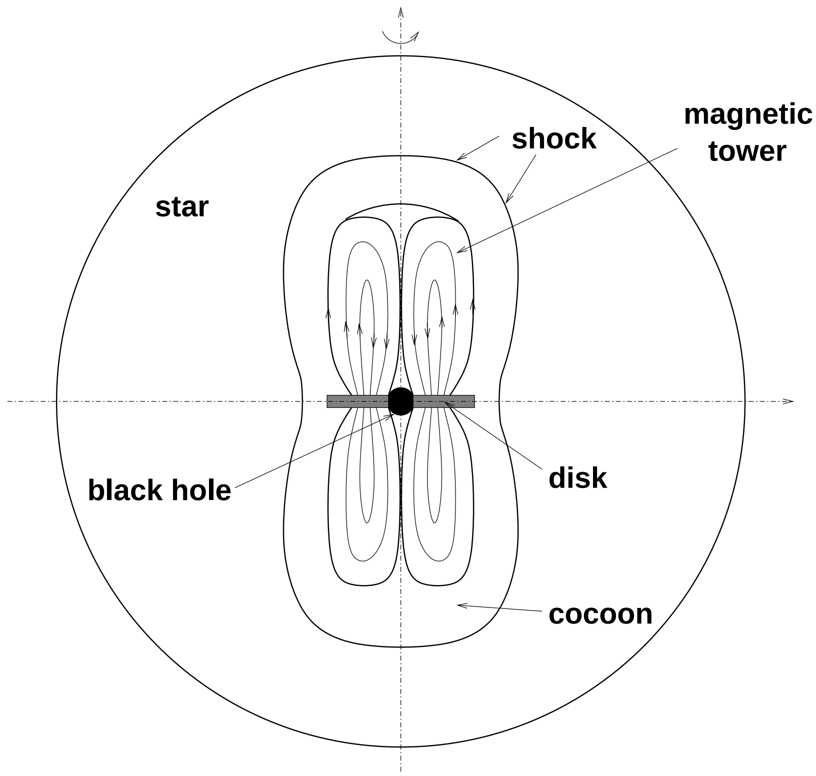


Why Are Magnetic Towers Attractive?

Advantages of the Magnetic Tower Model:

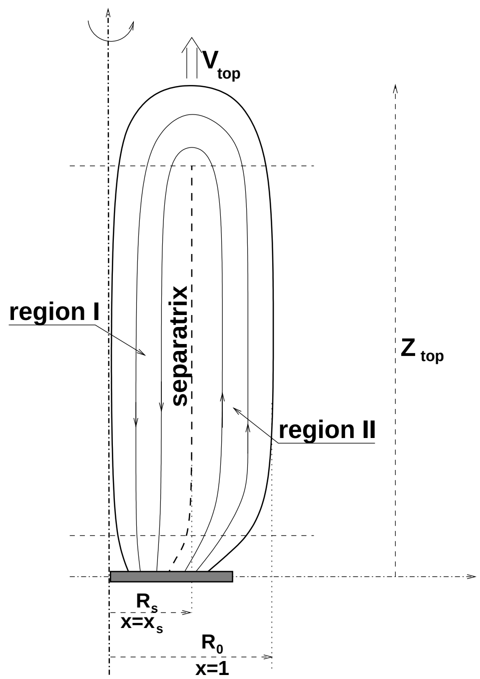
- Dense stellar envelope provides the most natural environment for magnetic tower collimation.
- Magnetic field produced by disk dynamo naturally has closed geometry: flux loops emerge from the disk and come back to the disk.
- Closed field lines inhibit contamination from the surrounding stellar material; magnetic tower just pushes the gas aside
  $\Rightarrow$ **low baryon loading!**

Magnetic Tower Inside a Star: Basic Model



- Unperturbed stellar pressure is negligible: $V_{\rm top} \gg c_{s*}$.
- External pressure is replaced by inertia of stellar gas.
- Tower drives a strong hydrodynamic shock is a magnetic piston.
- A hot cocoon confines the tower: $B^2_{\rm tower} = P_{\rm cocoon}$
- 3 input parameters: Ψ, $\Delta\Omega$, $P_{\rm ext}$ $\quad\Rightarrow\quad$ Ψ, $\Delta\Omega$, ρ_0

Magnetic Tower Inside a Star: Basic Estimates



$$V_{\rm top} \sim \Delta\Omega R_0 \quad \text{and} \quad V_{\rm top} \sim V_A \equiv \frac{B_0}{\sqrt{4\pi\rho_0}} \;\Rightarrow$$

$$R_0 \sim \left(\frac{\Psi}{\Delta\Omega}\right)^{1/3} \rho_0^{-1/6}$$

Estimates:

$$R_0 \;\sim\; 3\, R_d \left(\frac{B_{d,15}}{R_{d,6.5}\Delta\Omega_{3.5}\sqrt{\rho_{0,6}}}\right)^{1/3};$$

$$B_0 \;\sim\; 0.1\, B_{\rm d} \left(\frac{R_{d,6.5}\Delta\Omega_{3.5}}{B_{\rm d,15}}\right)^{2/3} \rho_{0,6}^{1/3};$$

$$V_{\rm top} \;\sim\; 3\cdot 10^{10}\,{\rm cm/sec}\; B_{d,15}^{1/3}\, R_{d,6.5}^{2/3}\, \Delta\Omega_{3.5}^{2/3}\, \rho_{0,6}^{-1/6}.$$

Magnetic Tower Inside a Star: Mathematical Model

- Axisymmetric Magnetic Field:

$$\mathbf{B}(R,z) = \mathbf{B}_{\rm pol} + B_\phi \hat{\phi} = \frac{1}{R}\,[\nabla\Psi \times \hat{\phi}] + \frac{I}{R}\,\hat{\phi}$$

- Force-free Grad–Shafranov equation:

$$R\,\partial_R\,(\frac{1}{R}\,\Psi_R) = -\,I I'(\Psi)$$

- Boundary Conditions:

$$\begin{aligned} \Psi_I(R=0) &= \Psi_{II}(R=R_0) = 1 \\ \Psi_I(R_s) &= \Psi_{II}(R_s) = 0 \end{aligned}$$

- Separatrix Force-Balance Condition:

$$(B_\phi^2 + B_z^2)^I = (B_\phi^2 + B_z^2)^{II}\,, \qquad R = R_s\,.$$

- Poloidal Current and the Twist Angle:

$$\Delta\Phi(\Psi) = \Delta\Omega(\Psi)\,t = I(\Psi)\,\int\limits_\Psi \frac{dz}{R^2 B_z}$$

Magnetic Tower Inside a Star: Mathematical Model (Cont'd)

- Tower-Cocoon Pressure Balance:

$$\frac{B_\phi^2(R_0)}{8\pi} + \frac{B_z^2(R_0)}{8\pi} = P_{\rm cocoon}\,.$$

- Total Vertical Magnetic Stress on Tower Top:

$$F_z = 2\pi \int\limits_0^{R_0} [\frac{B_\phi^2(R)}{8\pi} - \frac{B_z^2(R)}{8\pi}]\; RdR\,.$$

- Cocoon Pressure:

$$P_{\rm cocoon} = P_{\rm top} = \frac{F_z}{\pi R_0^2}$$

- Strong Shock Jump Condition:

$$P_{\rm cocoon} = \frac{4}{3}\,\rho_0 V_{\rm top}^2$$

Magnetic Tower Inside a Star: Analytical Example

Example: $I(\Psi) = \mu \sqrt{\Psi - \Psi^2}$

— linear ODE, eigen-value problem for μ.

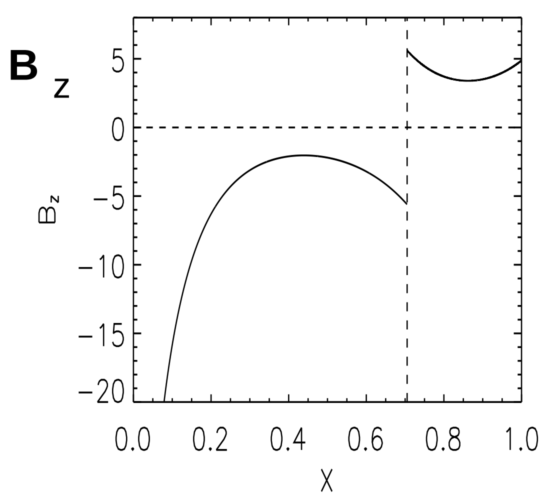
Solution in terms of modified Bessel functions:

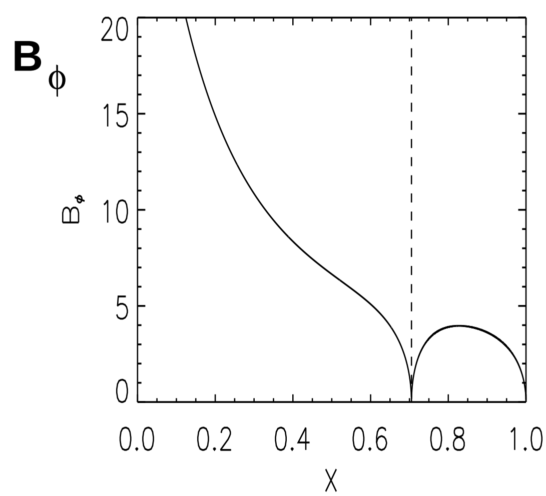
$$\psi_I(x) \;=\; \frac{1}{2} + a_1 x I_1(\mu x) + b_1 x K_1(\mu x)\,,$$
$$\psi_{II}(x) \;=\; \frac{1}{2} + a_2 x I_1(\mu x) + b_2 x K_1(\mu x)\,.$$

where a_1, b_1, a_2, b_2 are determined by boundary conditions.

Final values of parameters:

$$\mu \simeq 6.7 \qquad x_s \simeq 0.7 \qquad V_{\rm top} \simeq 3\,V_A$$





Transition to Relativistic Regime and the Final Opening Angle

- Tower radius increases as the tower grows and the surrounding density drops: $R_0 \sim \rho_0^{-1/6}$.

- The expansion velocity increases as $V_{\rm top} \sim \Delta\Omega R_0$.

- At some critical height z_c, density drops to a value ρ_c at which $V_{\rm top} = c \qquad \Rightarrow$
  transition to relativistic expansion !

- For $B_d = 10^{15}$ G, $R_d = 3 \cdot 10^6$ cm, $\Delta\Omega = 3 \cdot 10^3 \ {\rm sec}^{-1}$:
  $\rho_c \simeq 10^6 \ {\rm g/cm}^3 \qquad z_c \simeq 10^8$ cm.

- Relativistic Magnetic Tower $\quad \longrightarrow \quad$ Future Research.

- Insight from relativistic hydro simulations *(Zhang et al. 2003)*: collimation by the cocoon and recollimation shocks.

- Opening angle: $\Delta\theta \sim R_0/z_c \sim 0.1$?

SUMMARY

CONCLUSION:

Magnetic Tower inside a Collapsar Provides an Attractive Mechanism for the Formation of a Narrow, Baryon-Poor Channel through the Star.

TO BE DONE:

- Generalization to Relativistic Regime
- Numerical MHD Simulations
- Role of Instabilities and Magnetic Dissipation