Development of a General Relativistic Particle-in-Cell Code

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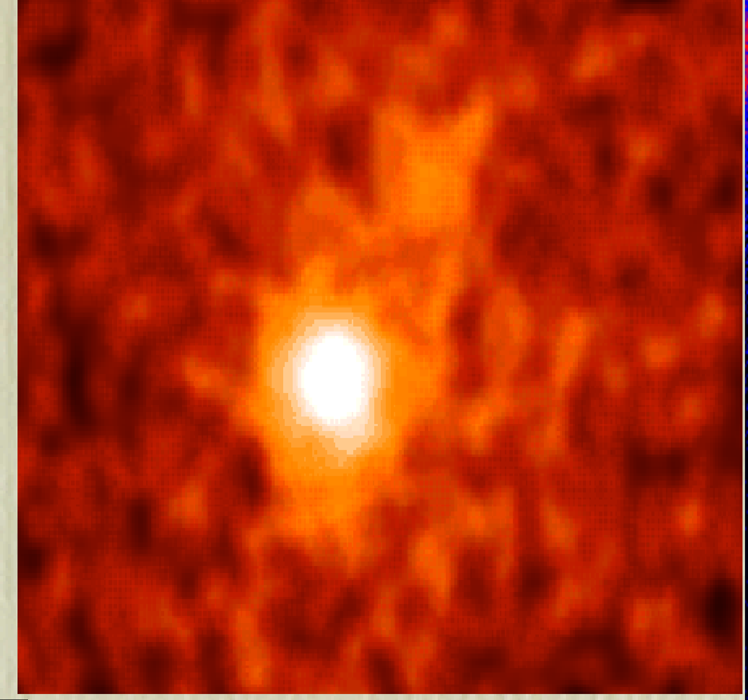
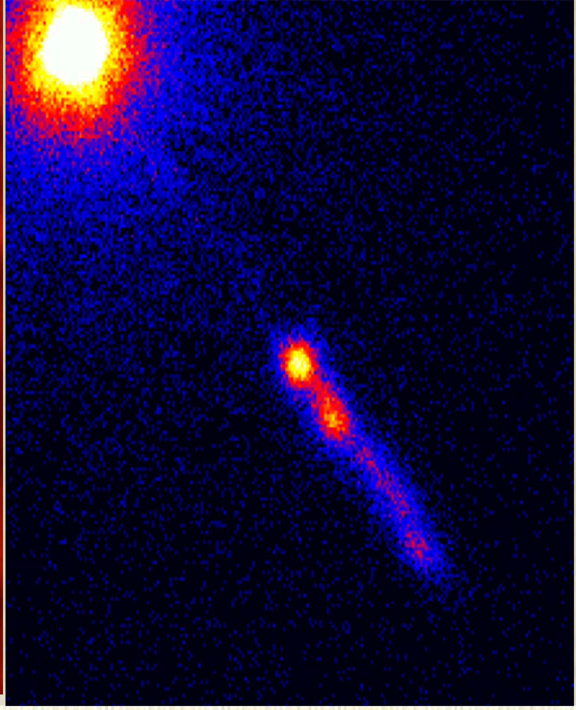
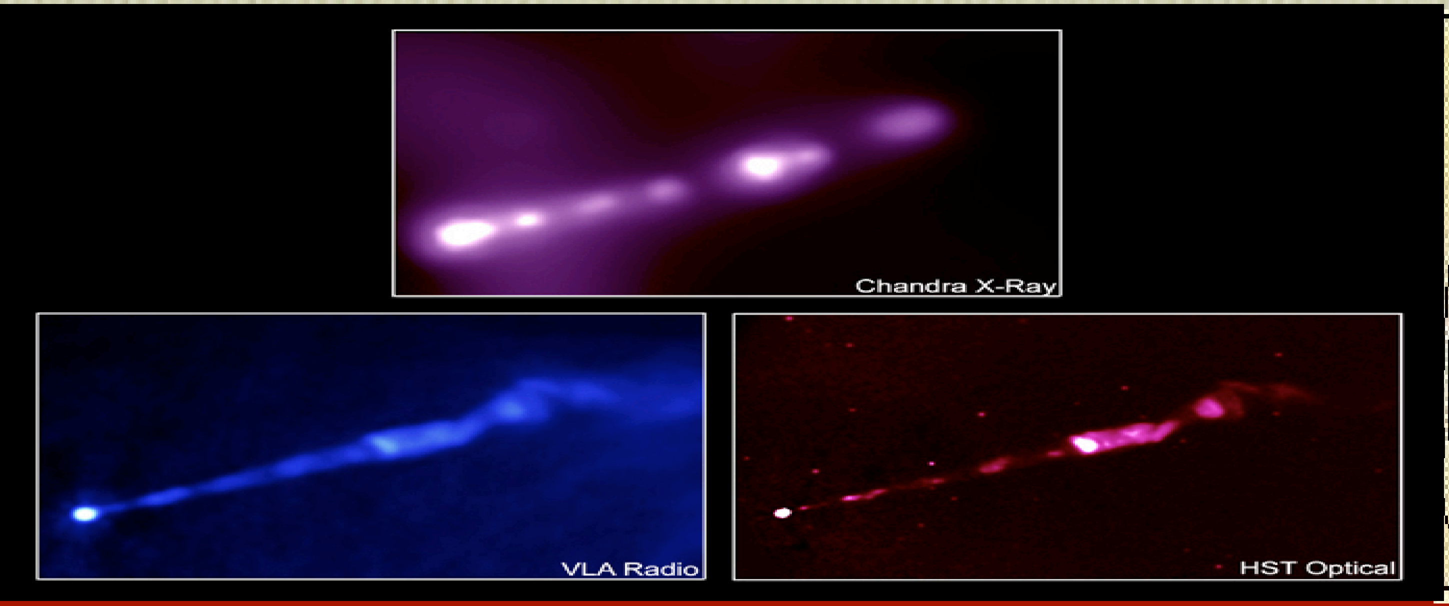
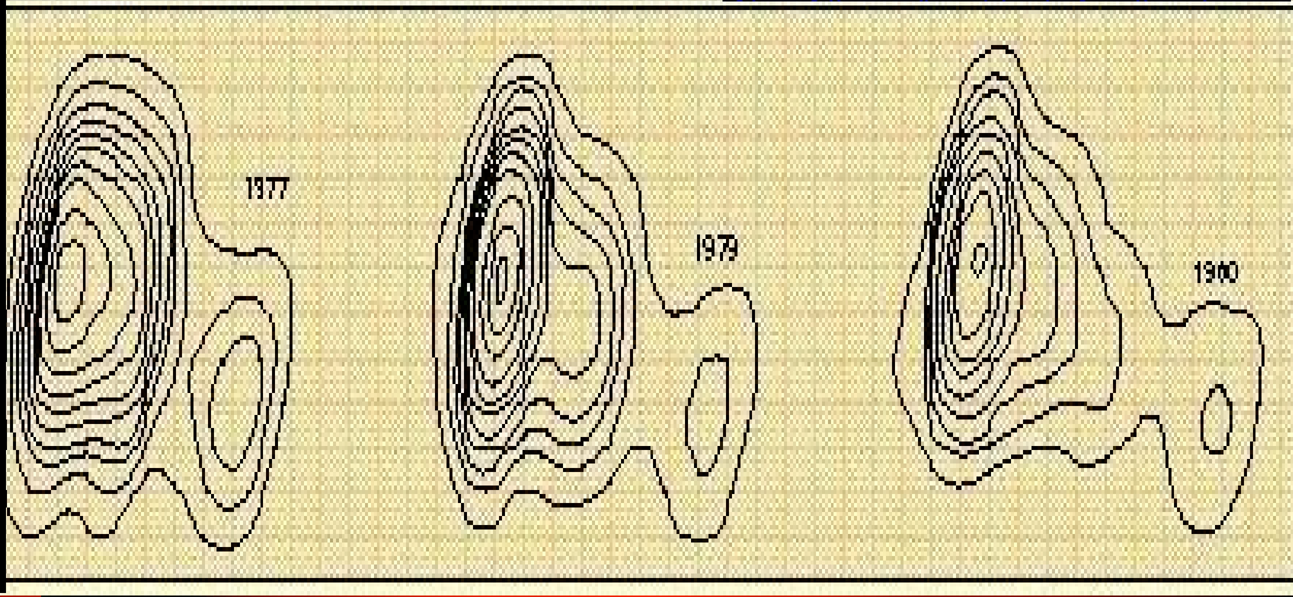
Observational Data

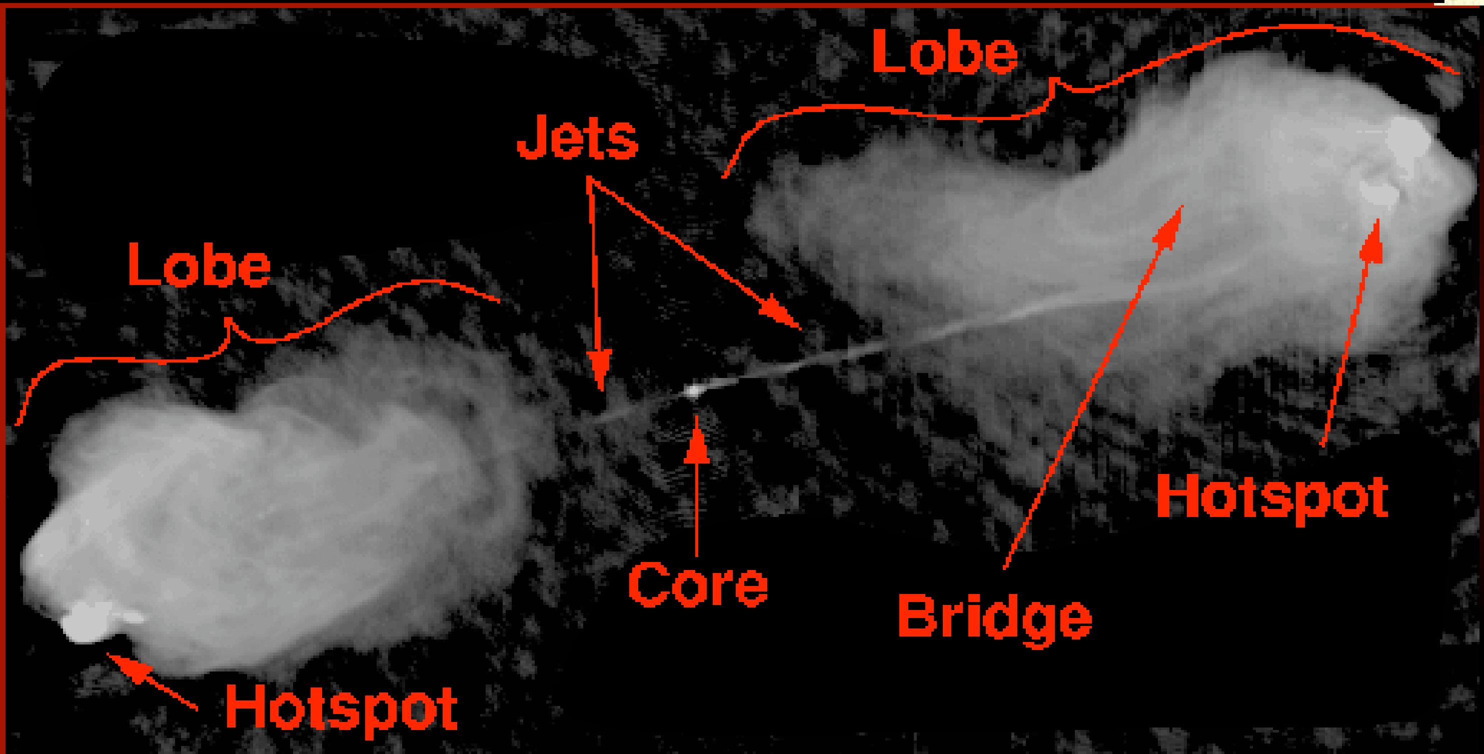
(X-ray) NASA/CXC/MIT/H. Marshall et al.; (Optical) NASA/STScI/UMBC/E. Perlman et al.); (Radio) NSF/NRAO/VLA)

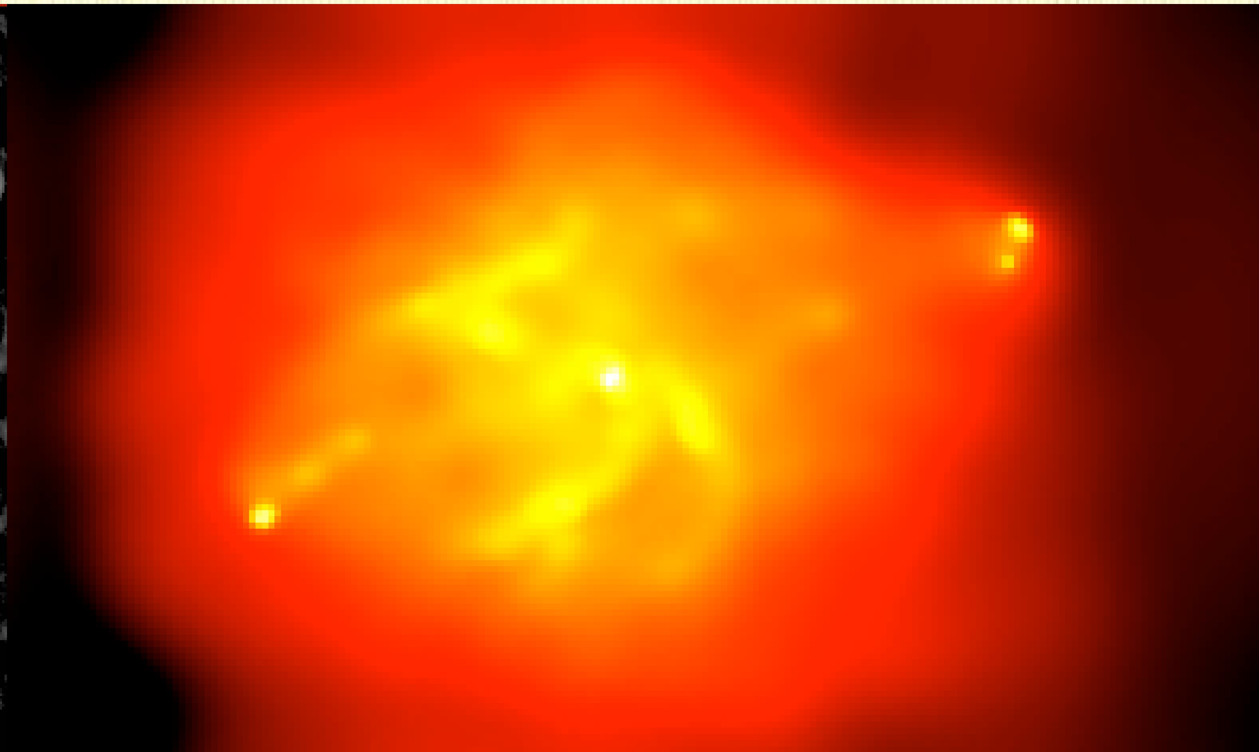












Relativistic PIC Jet/ Disk Simulation

- Plasma Accelerator
  - OSIRIS (Fonseca 2002)
- TRISTAN
  - O. Buneman and K. Nishikawa
- Relativistic Jet Simulation
  - Nishikawa et al 1996-2005
- Relativistic Disk Simulation
  - Lovelace 2004

GRPIC Considerations

Pros

- Particle motion is self-consistent.
- Dynamics of charged particle separation
- The major alternative, GRMHD, is a fluid approximation. Mainly uses “frozen in” condition and ideal fluid.

Cons

- Computationally intensive
- Dependence on initial conditions and instability growth
- Frequency scales

Accrete3D (GRPIC)

- General relativistic extension of relativistic particle-in-cell code
- Tensor form of Maxwell's equations
- Tensor form of Newton-Lorentz equation
- Kerr metric

General Relativistic Equations

Maxwell's Equations

$$F^{\alpha\beta}_{;\beta} = \frac{4\pi}{c} J^{\alpha} \qquad F_{\alpha\beta;\gamma} + F_{\beta\gamma;\alpha} + F_{\gamma\alpha;\beta} = 0$$

Equations of Motion

$$m\left(\frac{d^2x^\alpha}{d\tau^2} + \Gamma^\alpha_{rs}\frac{dx^r}{d\tau}\frac{dx^s}{d\tau}\right) = qF^\alpha_\beta u^\beta$$

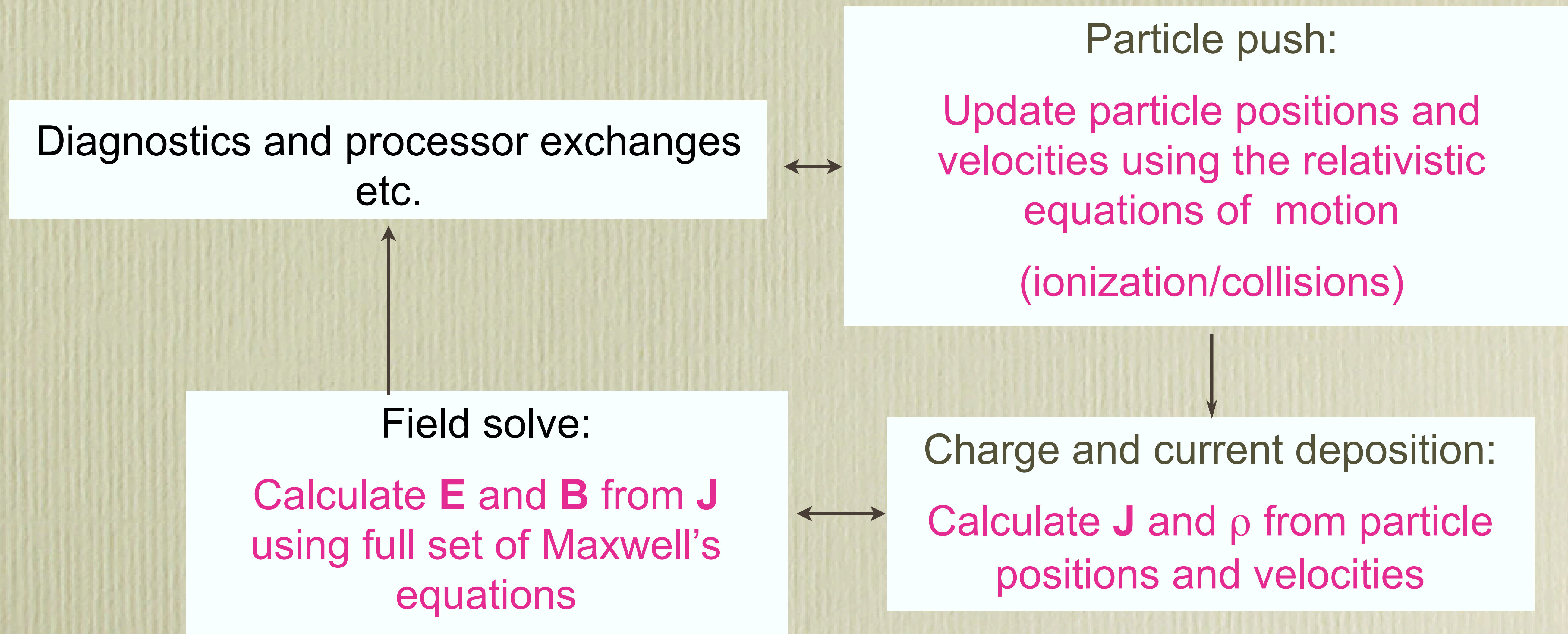
Kerr Metric

$$ds^2 = dx^2 + dy^2 + dz^2 - c^2dt^2 + \frac{2mr^3k^2}{r^4 + a^2z^2}$$

$$k(r^2 + a^2)r = r^2(xdx + ydy) + ar(xdy - ydx) + (r^2 + a^2)(zdz + rdt)$$

$$R^2 = x^2 + y^2 + z^2 \qquad r^4 - (R^2 - a^2)r^2 - a^2z^2 = 0$$

Particle-in-cell (PIC) code flow chart



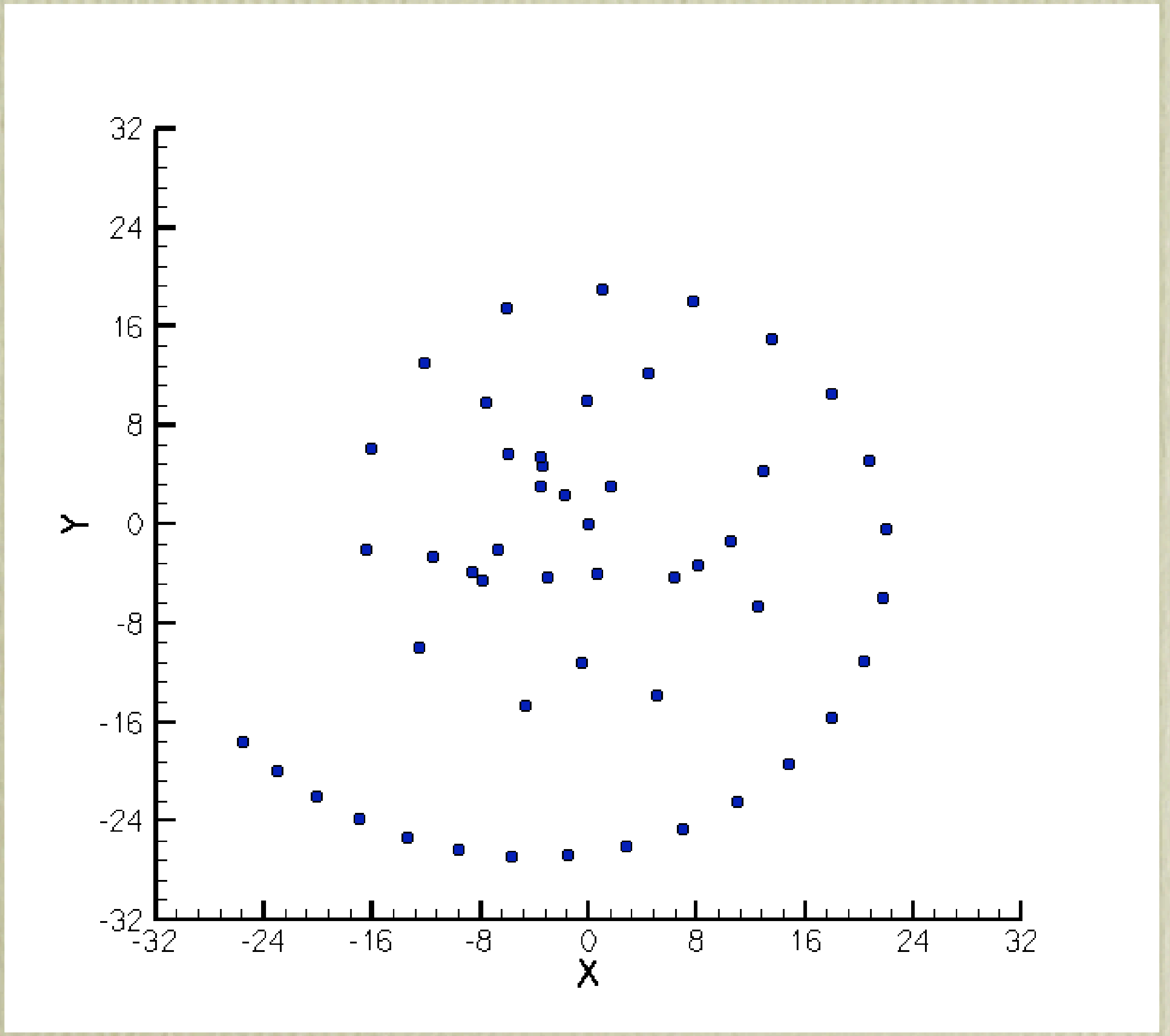
Particle Push

- Fifth Order Runge Kutta
- Adaptive Step Size
- Newton-Lorentz equation

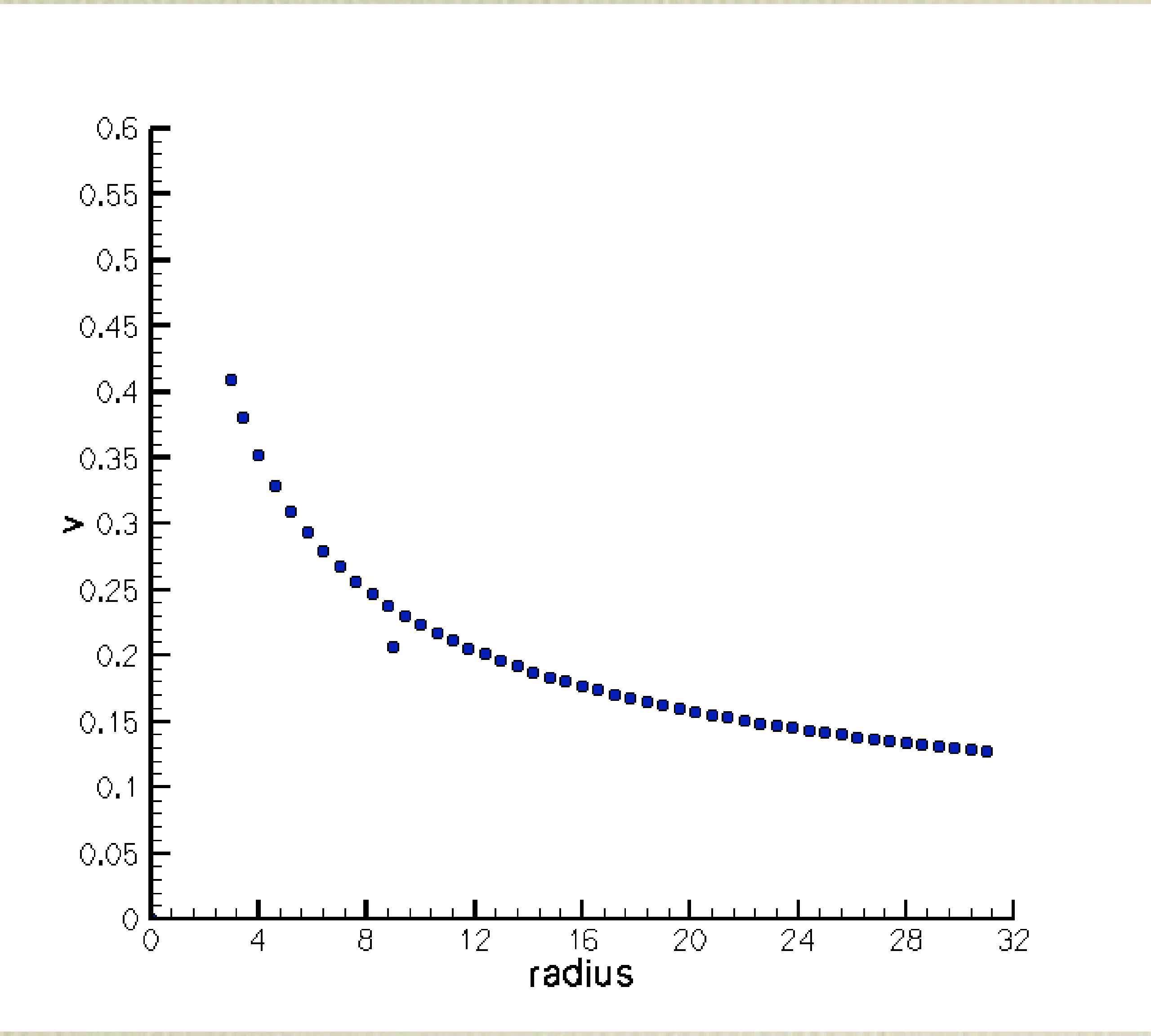
$$m\left(\frac{d^2x^\alpha}{d\tau^2} + \Gamma^\alpha_{rs}\frac{dx^r}{d\tau}\frac{dx^s}{d\tau}\right) = qF^\alpha_\beta u^\beta$$

$$u^\alpha = \frac{dx^\alpha}{d\tau} = \frac{dt}{d\tau}\frac{dx^\alpha}{dt} = \gamma v^\alpha$$

Orbit Test

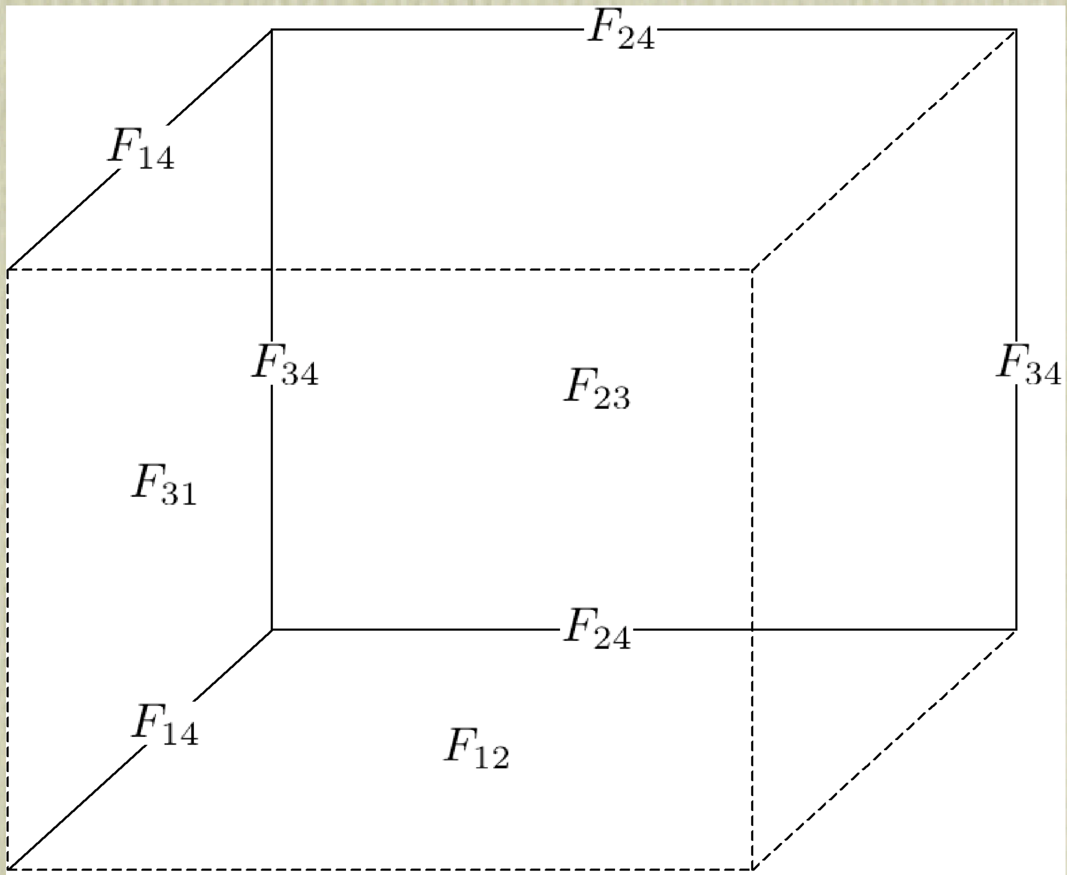


32
24
16
8
0
-8
-16
-24
-32
Y
-32
-24
-16
-8
0
8
16
24
32
X



0.6
0.55
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
v
0
4
8
12
16
20
24
28
32
radius

Yee Lattice



Magnetic Field

Classical

$$\frac{\partial \vec{B}}{\partial t} = -c\vec{\nabla} \times \vec{E}$$

General Relativity

$$F_{\alpha\beta;4} = -(F_{\beta 4;\alpha} + F_{4\alpha;\beta})$$

Magnetic Field

Classical

$$\frac{\partial \vec{B}}{\partial t} = -c\vec{\nabla} \times \vec{E}$$

General Relativity

$$F_{\alpha\beta,\gamma} + F_{\beta\gamma,\alpha} + F_{\gamma\alpha,\beta} = 0$$

Magnetic Field

Classical

$$\frac{\partial \vec{B}}{\partial t} = -c\vec{\nabla} \times \vec{E}$$

General Relativity

$$F_{\alpha\beta,\gamma} + F_{\beta\gamma,\alpha} + F_{\gamma\alpha,\beta} = 0$$

Code

$$F_{\alpha\beta}^{new} = F_{\alpha\beta}^{old} - \left(\frac{\partial F_{\beta 4}}{\partial x^{\alpha}} + \frac{\partial F_{4\alpha}}{\partial x^{\beta}} \right) dx^4$$

Electric Field

Classical

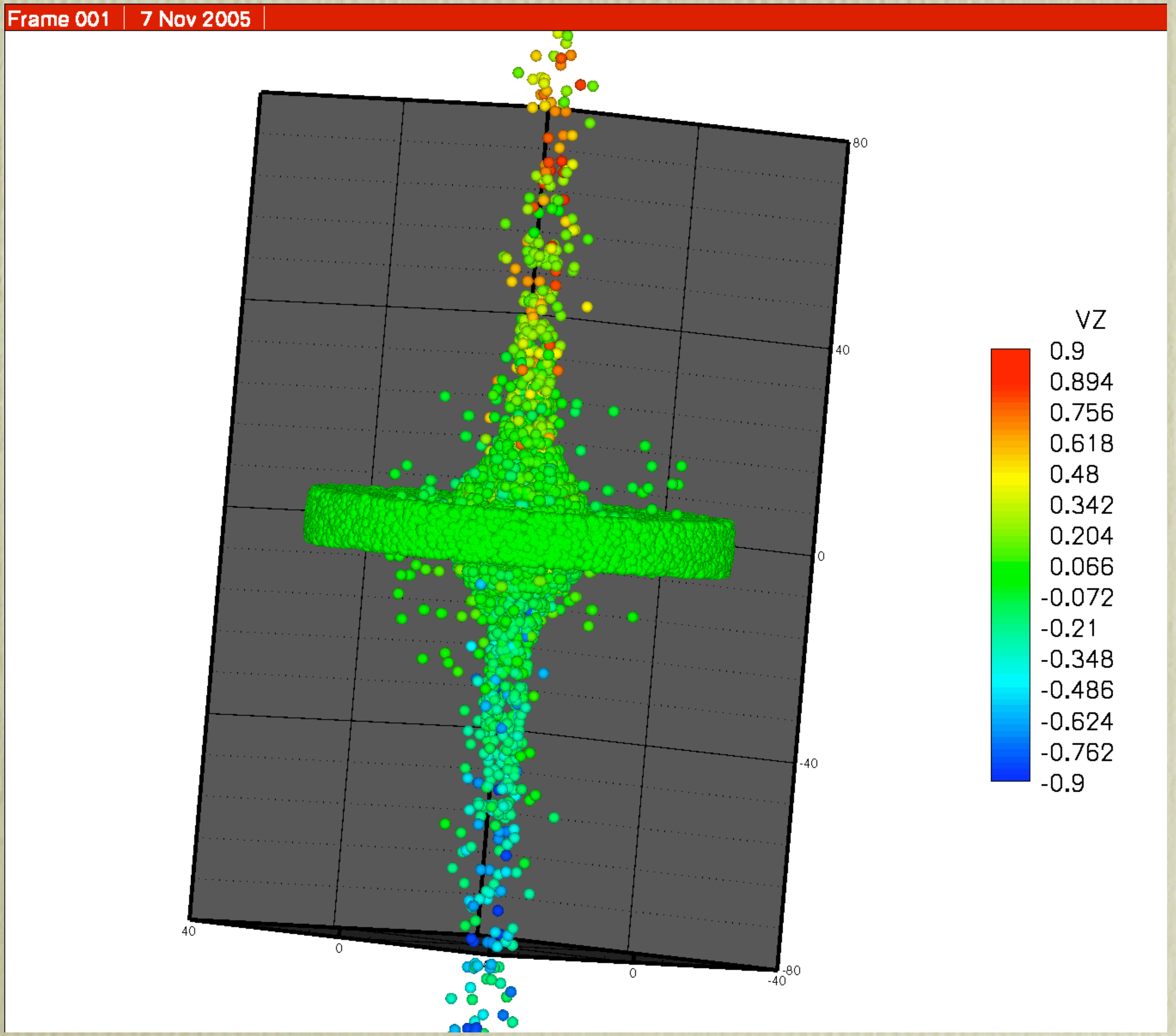
$$\frac{\partial \vec{E}}{\partial t} = c\vec{\nabla} \times \vec{B} - 4\pi \vec{J}$$

General Relativity

$$\frac{\partial F^{\alpha 4}}{\partial x^4} = \frac{4\pi}{c} J^\alpha - F^{\alpha\nu}_{\ ,\nu} - \Gamma^\alpha_{\gamma\beta} F^{\gamma\beta} - \Gamma^\beta_{\gamma\beta} F^{\alpha\gamma}$$

$$J^\alpha = \rho u^\alpha \qquad \rho = \gamma m n$$

Jet Simulation



B_z	-6 x 10^4 G
M_{bh}	3 X 10^8
Courant	.05
Disk Particles	400,000
Particle Mass	$1M_\odot$
Charge	10^{-5}
$\Delta t/\tau_s$	3.34
$\Delta x/r_s$	2.23

Considerations/Questions

- Increase grid resolution
- Incorporate MPI
- How strong are the outflows and what mechanism dominates the formation?
- What role does “orbital” instability play?
- What is the disk’s internal field structure?

Future Studies

- Disk/Jet Connection
- Magnetic field amplification and instabilities
- Energy transfer/conversion
- Radiation