Physical problems (microphysics) in relativistic plasma flows

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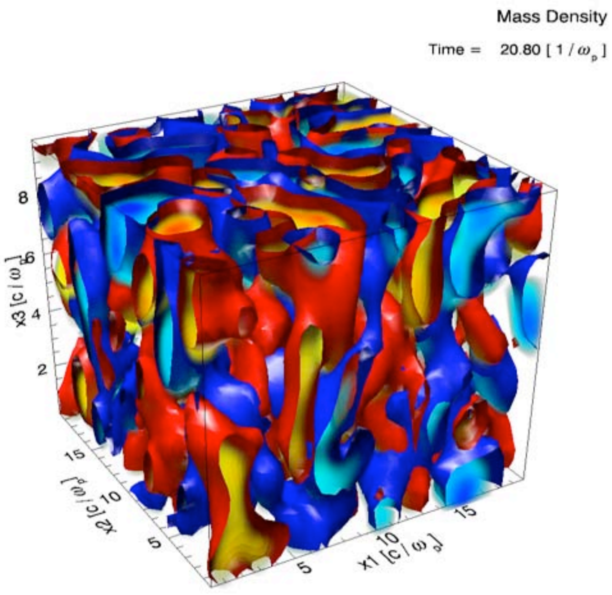
http://cfp.ist.utl.pt/golp/



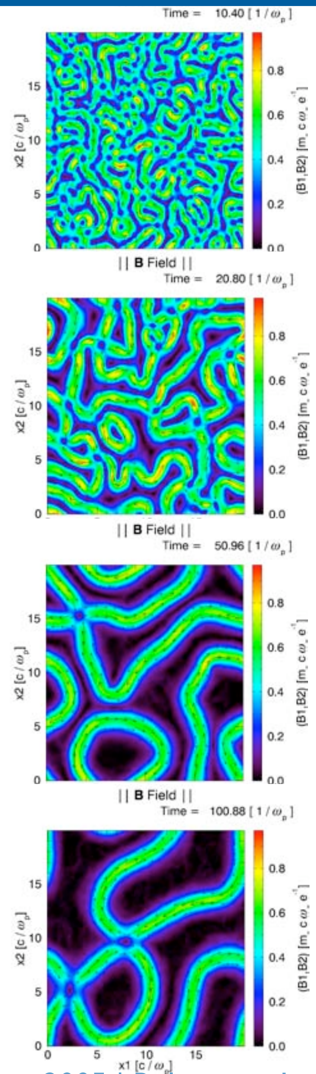
GoLP/CFP Instituto Superior Técnico

Collaborations

Massimiliano Fiore, Ricardo A. Fonseca, Michael Marti, Gianfranco Sorasio

Chuang Ren (now at U. Rochester), John Tonge, Michail Tzoufras, Warren B. Mori

Mikhail Medvedev



Outline

- Particle-in-cell simulations
  - probing the microphysics on the time scale of the electron collective dynamics ($f(\Upsilon)$, ϵ_B, ϵ_i, ϵ_e)
- Collisionless plasma instabilities
  - Weibel and two-stream: electromagnetic beam plasma instability
- Sub-equipartition B-field generation (ϵ_B)
- Particle acceleration ($f(\Upsilon)$)
- Summary

Plasma instabilities in GRBs

- In the fireball model of GRBs, relativistic plasma shells collide/overtake each other (Re'em Sari, yesterday; e.g. Piran, 05 and references therein)

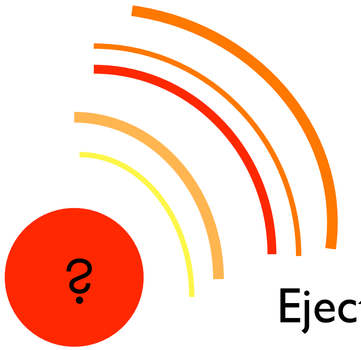
- Synchroton radiation indicates the presence of sub-equipartition magnetic fields: $\epsilon_{B\text{-field}}/\epsilon_{particles} \sim 10^{-3}$

- Weibel instability can be the mechanism responsible for B-field generation in this scenario (Medvedev and Loeb, 99)

Collision between internal shells
- internal shocks -

Conditions for plasma instabilities:
Relativistic shocks ?
Particle acceleration ?

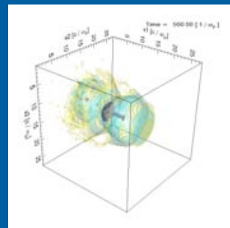
Central engine with
rapid variability



Collision between ejecta and ISM
- external shock -

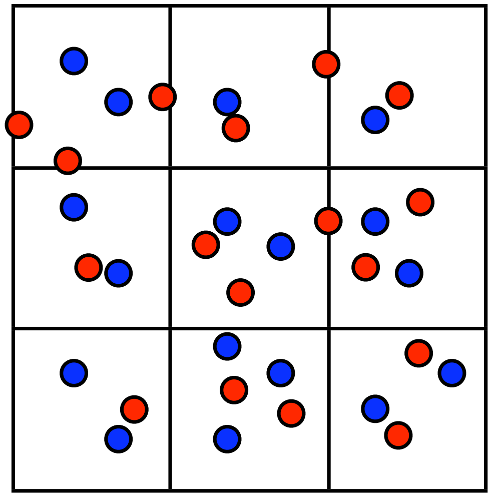
Ejecta of relativistic
plasma shells

Numerical simulations



Particle-in-cell simulations

Solving Maxwell's equations on a grid with self-consistent charges and currents due to charged particle dynamics



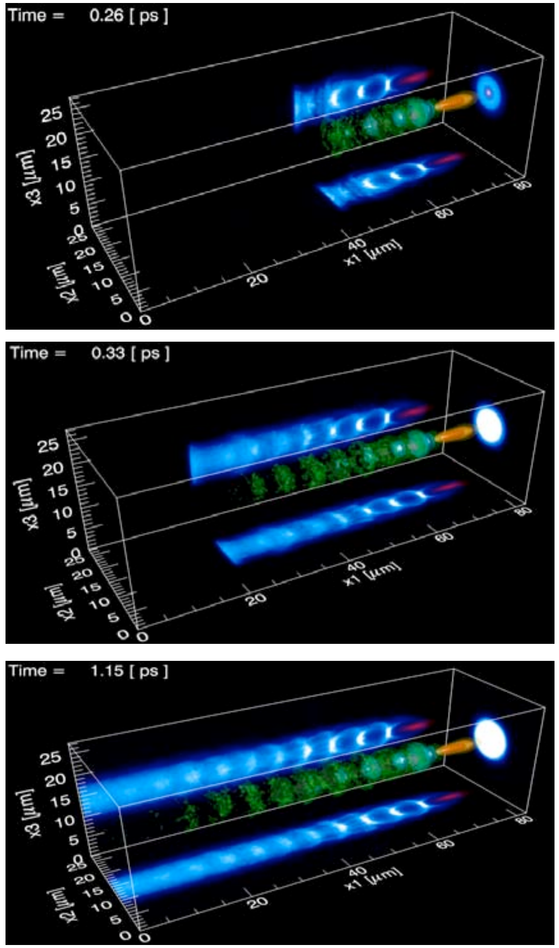
State-of-the-art

~ 10^9 particles

~ $(500)^3$ cells

RAM ~ 0.5 TByte
Run time: hours to months
Data/run ~ 1 TByte

One-to-one simulations of plasma based accelerators & cluster dynamics
Weibel/two stream instability in fast igniton, astrophysics

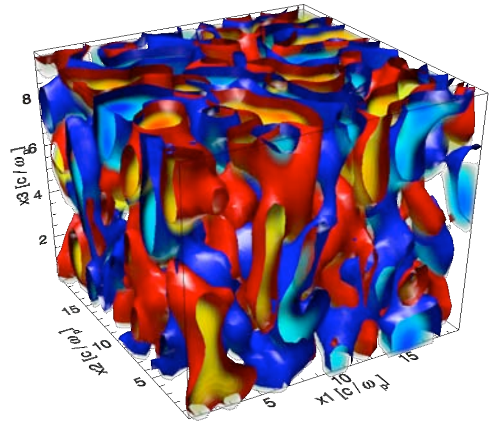


Particle-in-cell (PIC) - (Dawson, Buneman, 1960's)
Maxwell's equation solved on simulation grid
Particles pushed with Lorentz force

osiris 2.0

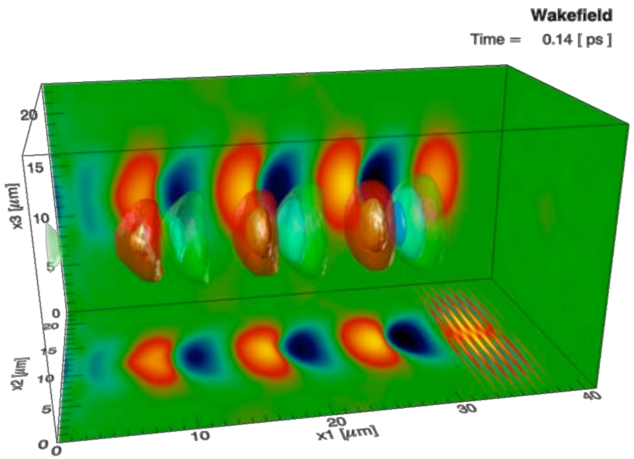
osiris.framework

- Massivelly parallel particle-in-cell code
- Visualization and data analysis infrastructure
- Developed by the *osiris.consortium*
  - ⇒ UCLA + IST + USC

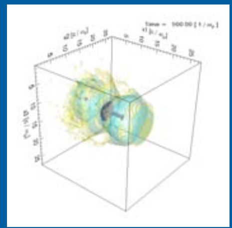


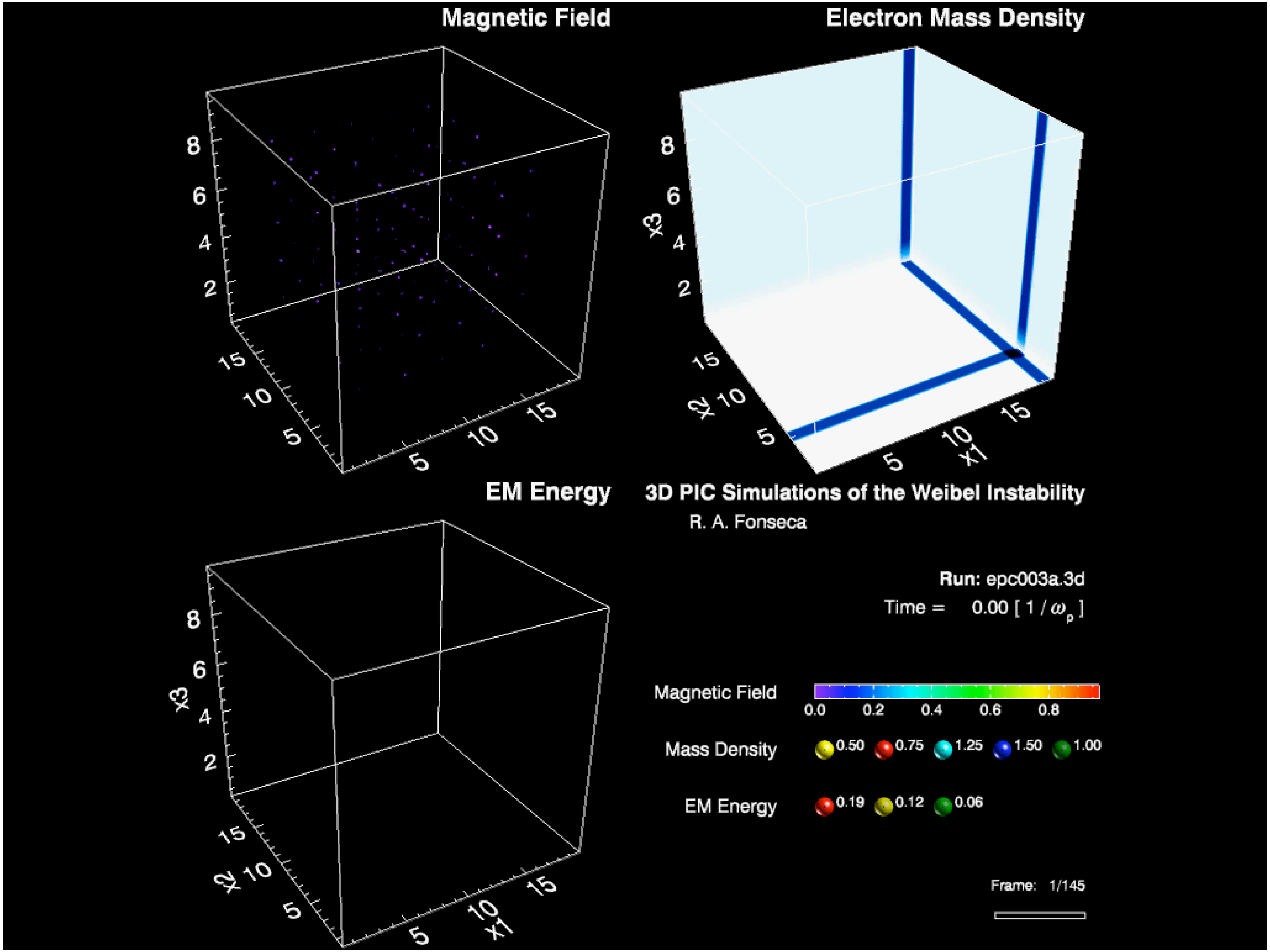
New in version 2.0

- Bessel beams
- Binary collisions module
- Impact and tunnel ionization (ADK model)
- Dynamic load balancing
- Parallel I/O



Collisionless plasma instabilities




Magnetic Field
Electron Mass Density
EM Energy
3D PIC Simulations of the Weibel Instability
R. A. Fonseca
Run: epc003a.3d
Time = 0.00 [1 / ω_p]
Magnetic Field
0.0 0.2 0.4 0.6 0.8
Mass Density
0.50 0.75 1.25 1.50 1.00
EM Energy
0.19 0.12 0.06
Frame: 1/145
x1
x2
x3

Collisionless instabilities of plasma streams

General theory: Watson, Bludman, Rosenbluth (1960)

Two-stream instability (longitudinal mode):

$\Gamma \sim (n_b/n_0)^{1/3}$

$k_{||} \sim \omega_{pe0}/v_b$

n_b, v_b - number density/velocity of plasma stream

n_0 - number density of background (stationary) plasma

Weibel/filamentation instability for e^-/e^+ (transverse mode):

$\Gamma \sim (n_b/n_0)^{1/2}$

$k_\perp \sim \omega_{pe0}/c \; (T \neq 0)$

Multidimensional electromagnetic beam-plasma instability will be a combination of transverse and longitudinal modes

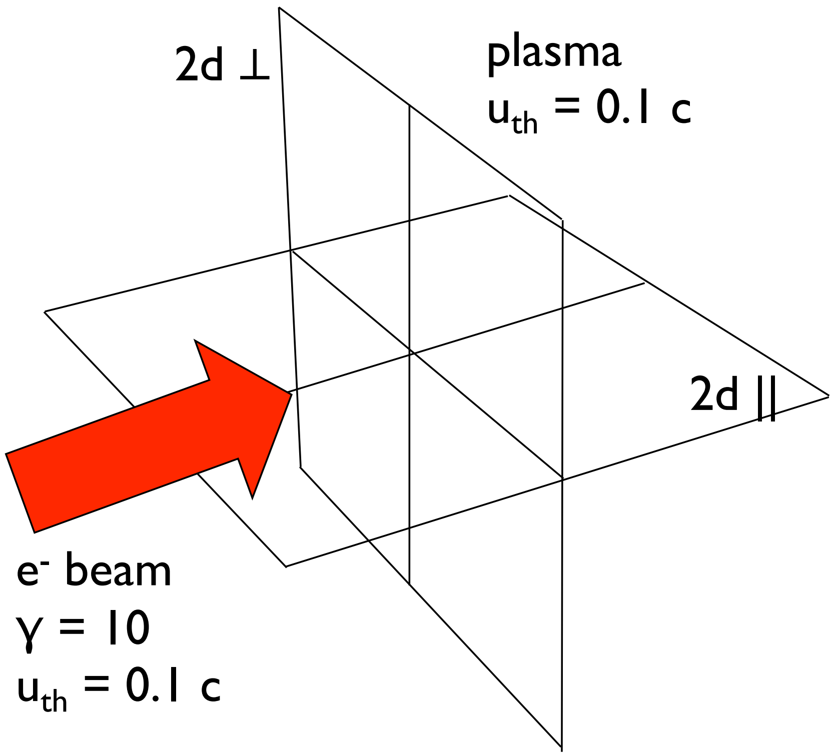
Relevant time and spatial scales

$$\mathrm{L} \approx c/\omega_{p0} \approx 5\,\mathrm{km}/\sqrt{\mathrm{n}_0[\mathrm{cm}^-3]}$$

$$\mathrm{T} \approx 1/\omega_{p0} \approx 20\,\mu\mathrm{s}/\sqrt{\mathrm{n}_0[\mathrm{cm}^-3]}$$

Three dimensional PIC simulations I

Osiris* and PARSEC**



$n_b/n_0 = 0.1$

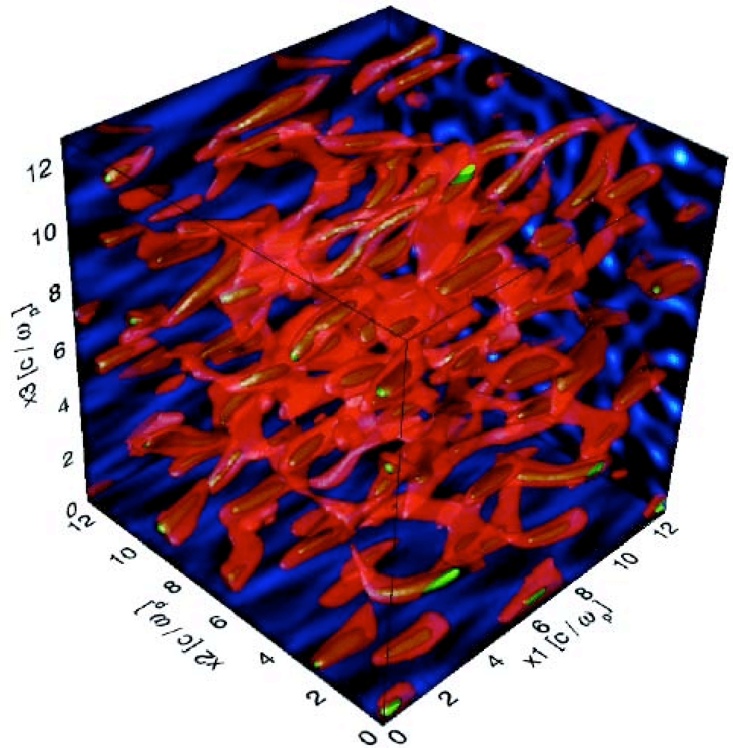
Periodic system
Current and charge neutral
Moving ions

- 3D runs
  - $(128)^3$ cells
  - $(12.8\ c/\omega_{p0})^3$
  - $\omega_{p0}t_{max} = 500$
  - 16 particles/(species×cell)
- 2D runs (|| and ⊥)
  - (512 × 128) cells
  - $(51.2 \times 12.8)\ (c/\omega_{p0})^2$
  - $\omega_{p0}t_{max} = 500$
  - 16 particles/(species×cell)

*R.A. Fonseca et al, LCNS 2002;
** J. Tonge, UCLA PhD Thesis 2002

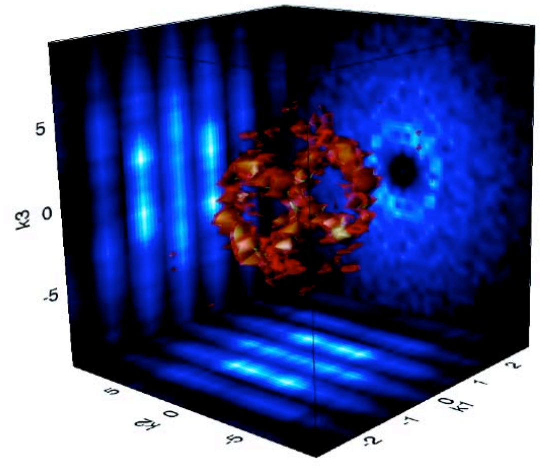
Three dimensional PIC simulations II

Beam density @ $t = 62.4/\omega_{pe0} = 7.5\ \Gamma_{weibel}$



Green = 1.15 n_{e0}; Red = 0.53 n_{e0}

FFT



Green = 40%; Red = 27.5%

Fastest growing mode

$k_{\parallel} \sim 1$ $k_{\perp} \sim 3$

$\theta_{tilt} \sim 18.4°$

Covariant fluid theory I

$$\frac{\partial T_i^{\alpha\beta}}{\partial x^\beta} = m_i \bar{n}_i F^\alpha$$ Covariant fluid equations $F^\alpha \equiv \text{Lorentz 4} - \text{force}$

$$T^{\alpha\beta} = p\eta^{\alpha\beta} + (p+\varepsilon)U^\alpha U^\beta$$ Energy-momentum stress tensor

3D representation

$$(\partial_t + \mathbf{v}\cdot\nabla)\mathbf{v} = \frac{qnc^2}{\gamma^2(p+\varepsilon)}\left[\mathbf{E} + \frac{\mathbf{v}}{c}\times\mathbf{B}\right] - \frac{qn}{\gamma^2(p+\varepsilon)}(\mathbf{v}\cdot\mathbf{E})\mathbf{v} - \frac{c^2}{\gamma^2(p+\varepsilon)}\left[\nabla p + \frac{\mathbf{v}}{c^2}\partial_t p\right]$$

+ Maxwell's equations

Key issue: relativistic equation of state p(T,n) and ϵ(T,n)

Covariant fluid theory II

$$U^\alpha = \int u^\alpha \frac{f(u)}{(1+\mathbf{u}\cdot\mathbf{u})^{1/2}} d\mathbf{u}$$

Fluid proper velocity

$$T^{\alpha\beta} = mc^2 \int u^\alpha u^\beta \frac{f(u)}{(1+\mathbf{u}\cdot\mathbf{u})^{1/2}} d\mathbf{u}$$

Energy-momentum stress tensor

In the rest frame of the fluid: $T^{11} = p \qquad T^{00} = \varepsilon$

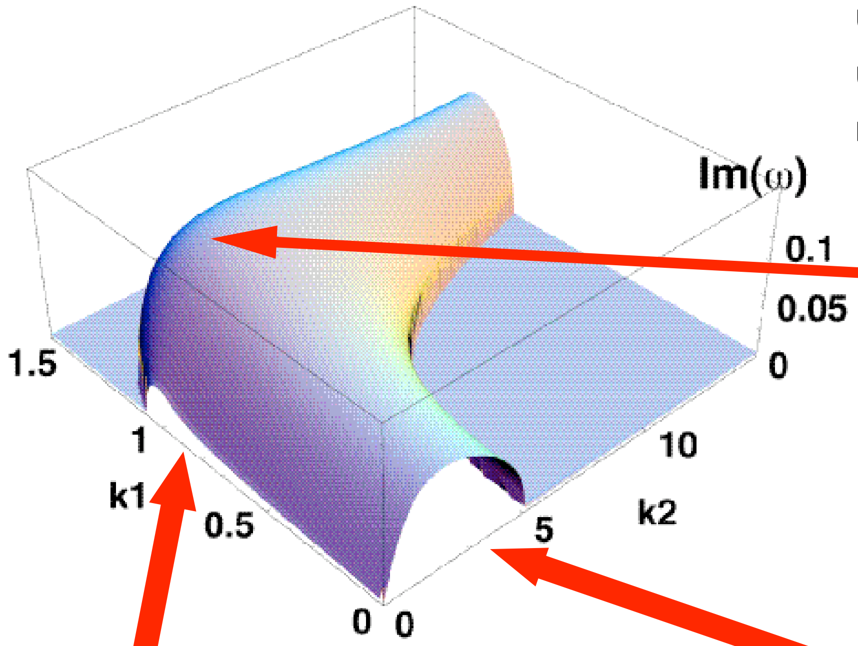
For a waterbag f(u) :

$$\frac{p}{n_0 mc^2} = \frac{1}{8u_0^3}\left\{u_0\tilde{n}\left(1+u_0^2\tilde{n}^2\right)^{1/2}\left(2u_0^2\tilde{n}^2-3\right)+3\,\mathrm{arcsinh}\left(u_0\tilde{n}\right)\right\} \qquad \tilde{n} = \left(\bar{n}/n_0\right)^{1/3}$$

$$\varepsilon = n_0 mc^2\left\{\tilde{n}+\frac{1}{2}u_0^2\tilde{n}^{5/3}\right\} - p$$

Tilted filamentation of relativistic electron streams I*

Multidimensional (2D) electromagnetic beam-plasma instability ($T \neq 0$)



$u_b = 10$
$u_0 = 0.22$
$n_b = 0.1\ n_0$

Fastest growing mode
$k_{1mx} \sim 1\ k_{2mx} \sim 3$

$\theta_{tilt} = \arctan(k_{1mx}/k_{2mx})$
$\theta_{tilt} \sim 18.4°$

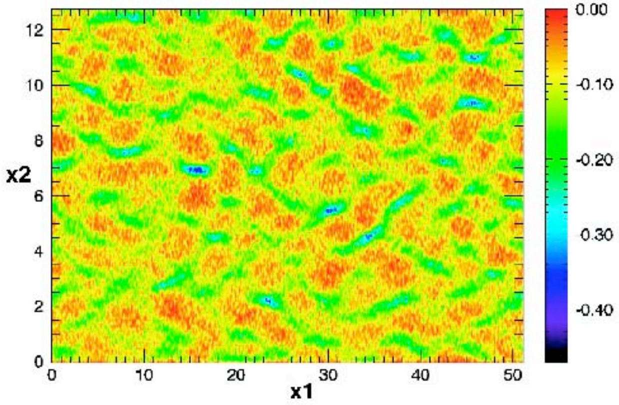
two-stream instability
(similar to fluid theory)

Weibel instability
(similar to waterbag kinetic)

*Silva et al, Bull. Am. Phys. Soc., 2001; Bret and Deutsch, Phys. Rev. Lett, 2004

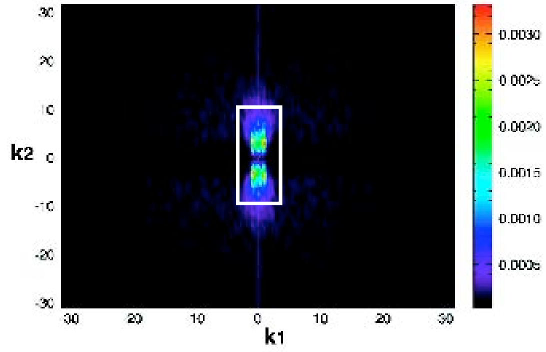
Comparison theory-simulations I

Electron beam spatial structure

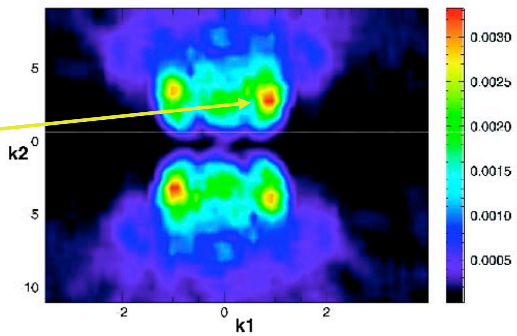


Beam

FFT



FFT (zoom)

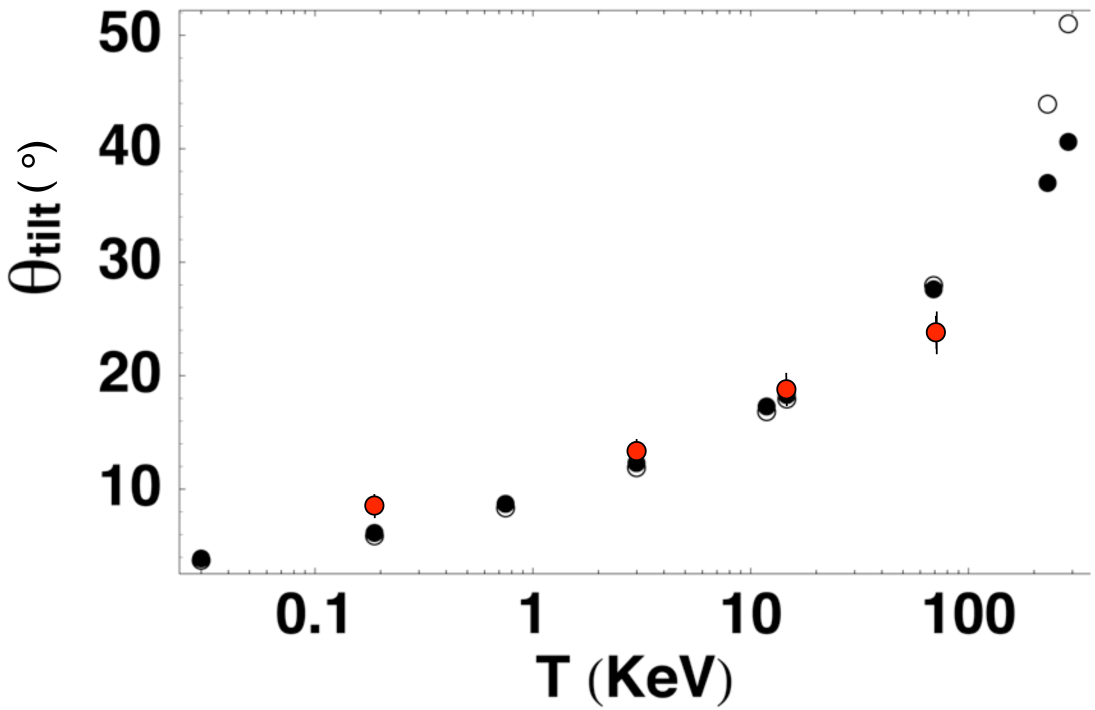


$k_{1mx} \sim 1, k_{2mx} \sim 3.0$

Typical spraying angle ~ 18°

2D || runs
(512 × 128) cells
(51.2 × 12.8) $(c/\omega_{p0})^2$
$\omega_{p0}t_{max}$= 500
16 particles/(species×cell)

Comparison theory-simulations II



$$\frac{T}{mc^2} = \int \mathbf{u}\cdot\mathbf{u}\frac{f(u)}{(1+\mathbf{u}\cdot\mathbf{u})^{1/2}}d\mathbf{u}$$

2D || runs
(512 × 128) cells
(51.2 × 12.8) $(c/\omega_{p0})^2$
$\omega_{p0}t_{max}$= 500
16 particles/(species×cell)

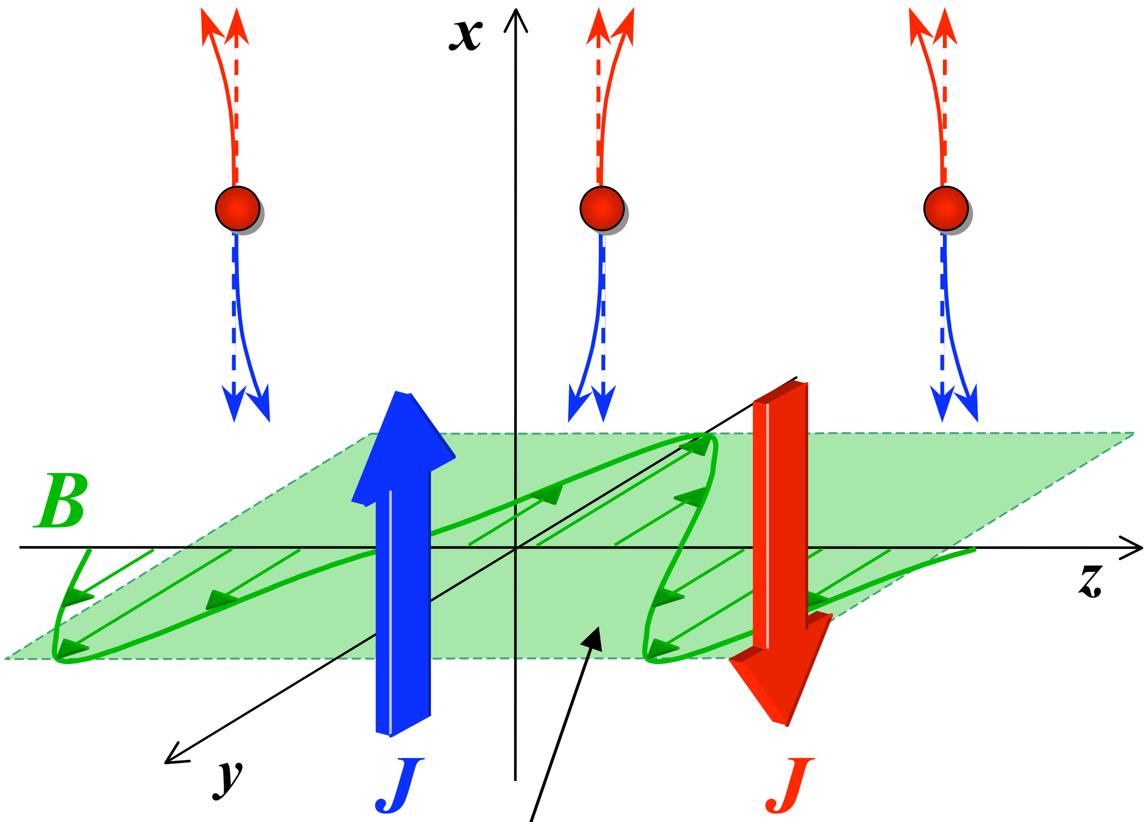
u_b = 10
n_b = 0.1 n_0 ○
n_b = 0.0125 n_0 •
Simulation n_b = 0.1 n_0 ●

Main consequence of tilted filamentation (for hot beams)
nonlinear evolution/interaction of filaments
beam spraying

Weibel instability*

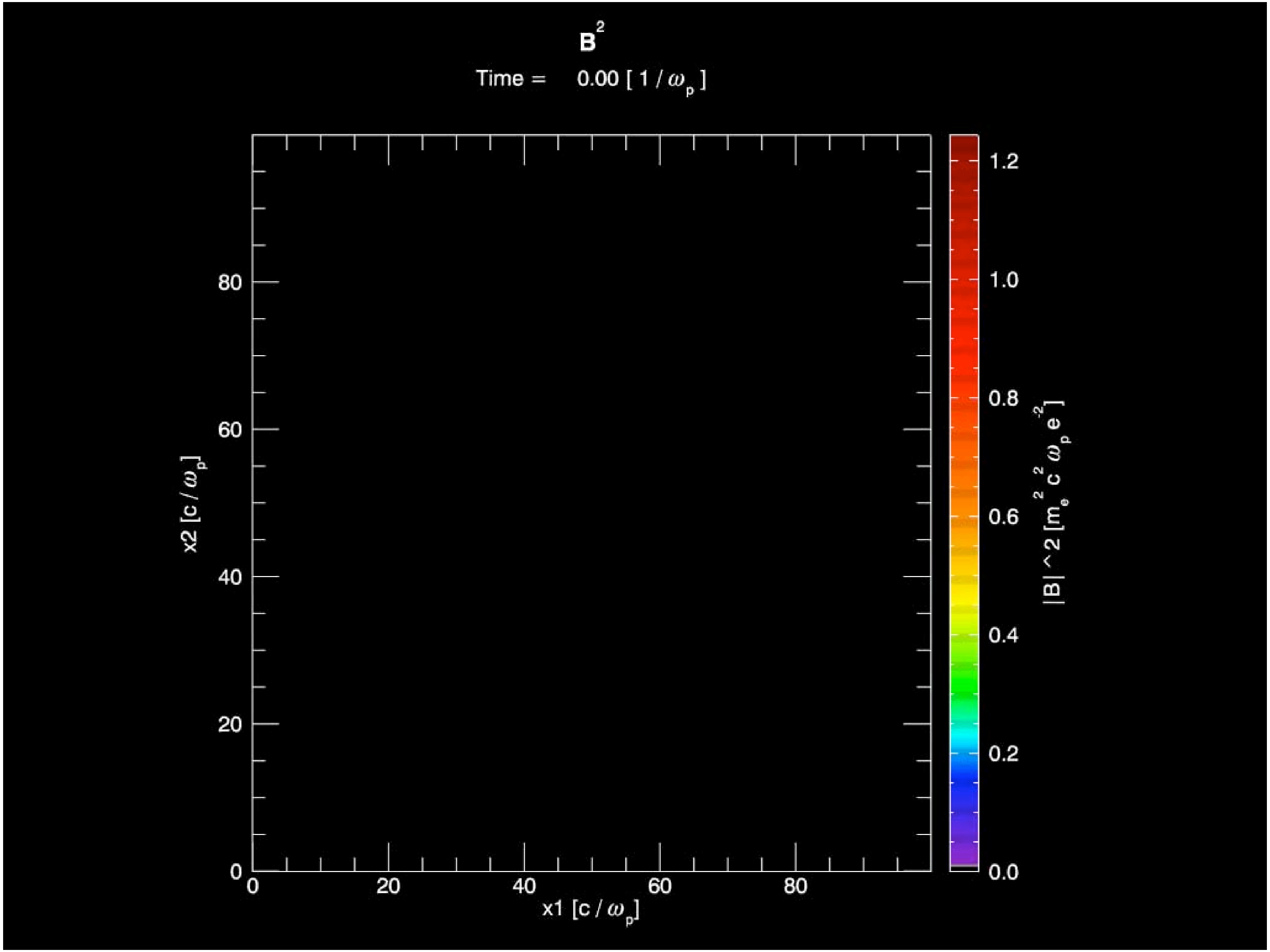
Fluctuation in the magnetic field leads to fluctuation in current density which in turn enhances magnetic field fluctuation

Free energy of particles converted to (mainly) quasi-static magnetic fields



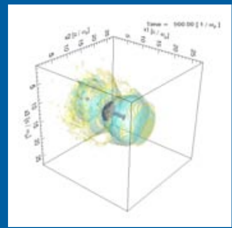
Collision plane of plasma shells

*E.S. Weibel, Phys. Rev. Lett. , 1959



B^2
Time = 0.00 [1 / ω_p]
x2 [c / ω_p]
x1 [c / ω_p]
|B| ^2 [m_e^2 c^2 ω_p e^-2]
0
20
40
60
80
0.0
0.2
0.4
0.6
0.8
1.0
1.2

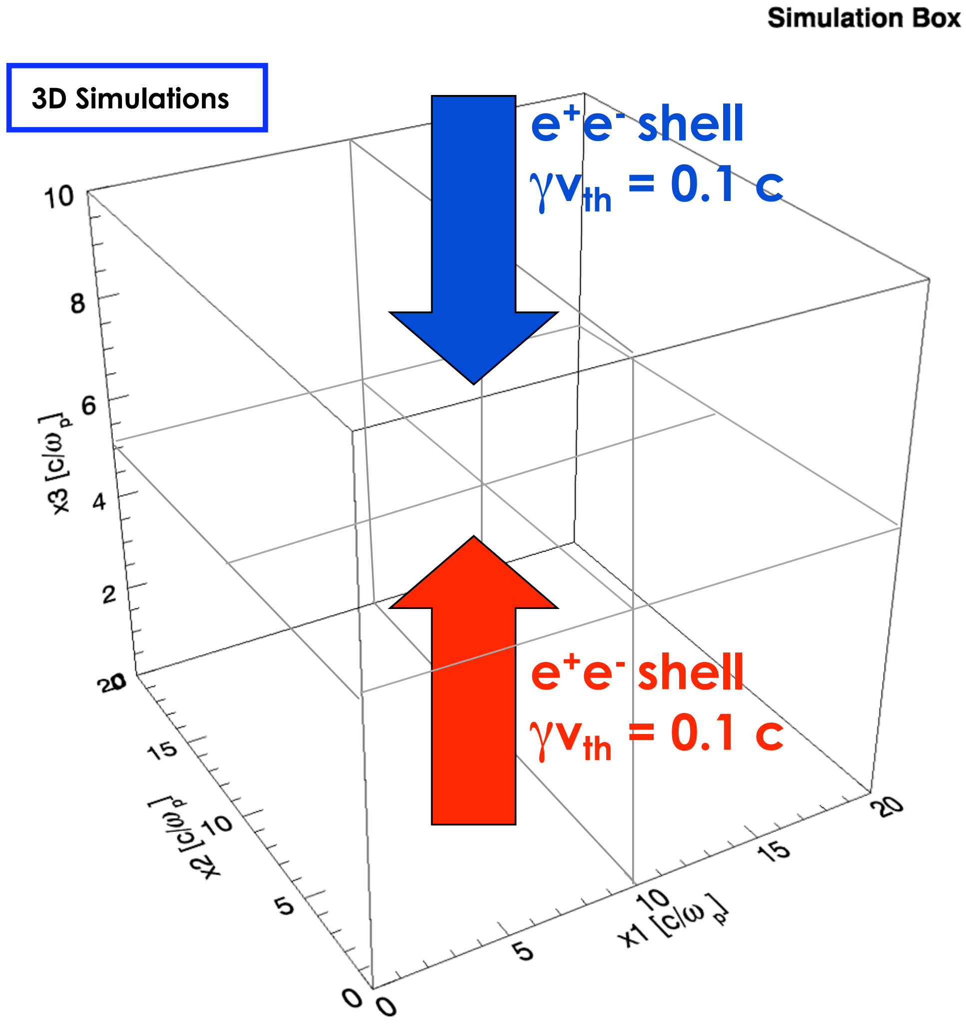
PIC Simulations



What have we learned so far from PIC simulations?

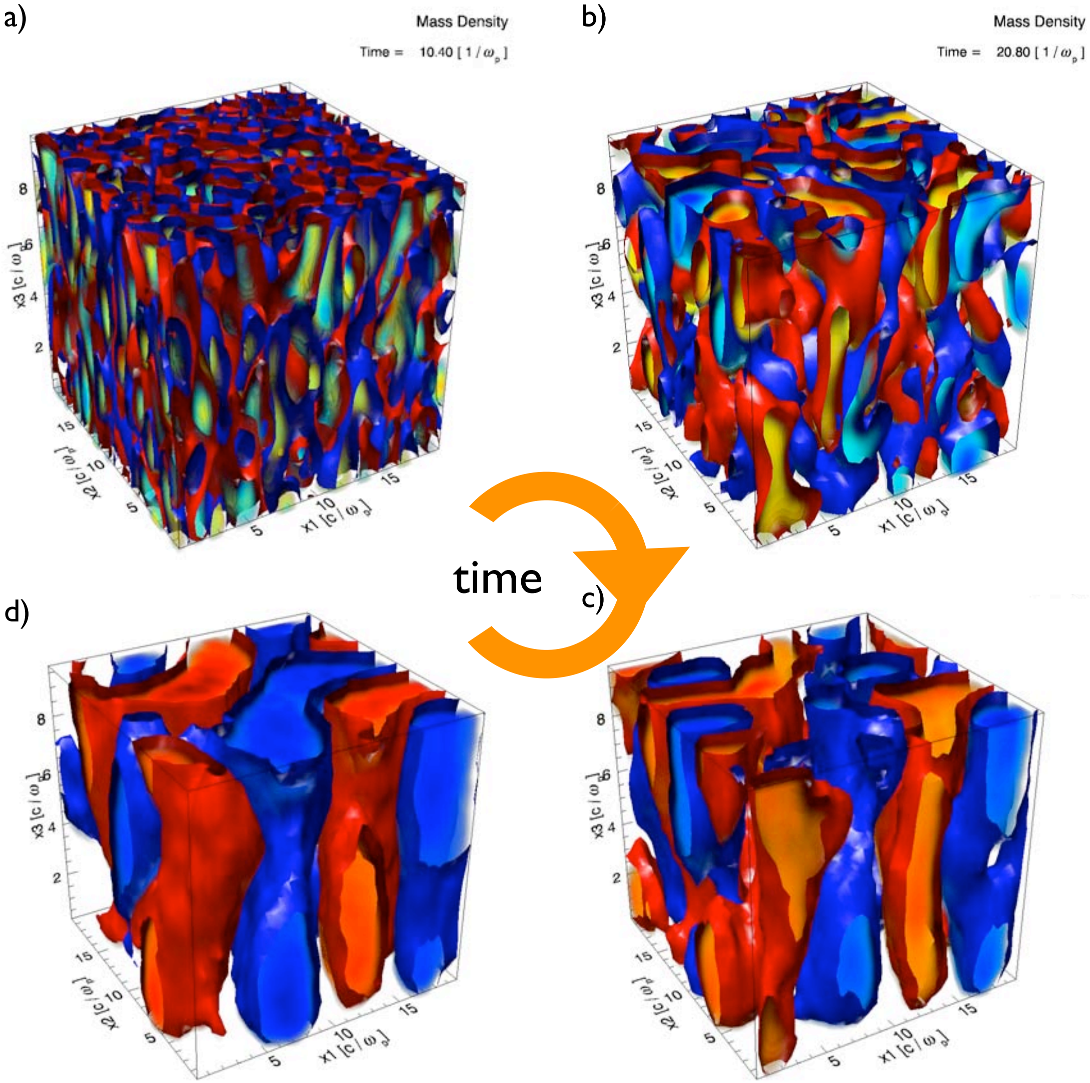
- Kazimura et al (1998); Gruzinov (2001); Silva et al (2003); Nishikawa et al (2003); Frederiksen et al (2004); Hededal and Nordlund (2005); Milosavljevic et al (2005)
  - Weibel seems to work (sub-equipartition B-fields) (e-e+ & very light ions)
  - Nonlinear dynamics of filaments leads to B-field evolution on a very slow time scale (much slower than $1/\omega_{p0}$)
  - Reconnection in nonlinear stage of instability leads to heating/acceleration
  - Radial space-charge fields associated with filaments play a role in particle acceleration/heating
  - Recent works are already analyzing synchrotron spectra measured in the simulations

Numerical set-up



- **3D simulations**
  200 x 200 x 100 cells
  (20 x 20 x 10 c^3/ω_p^3)
  2 particles per cell species

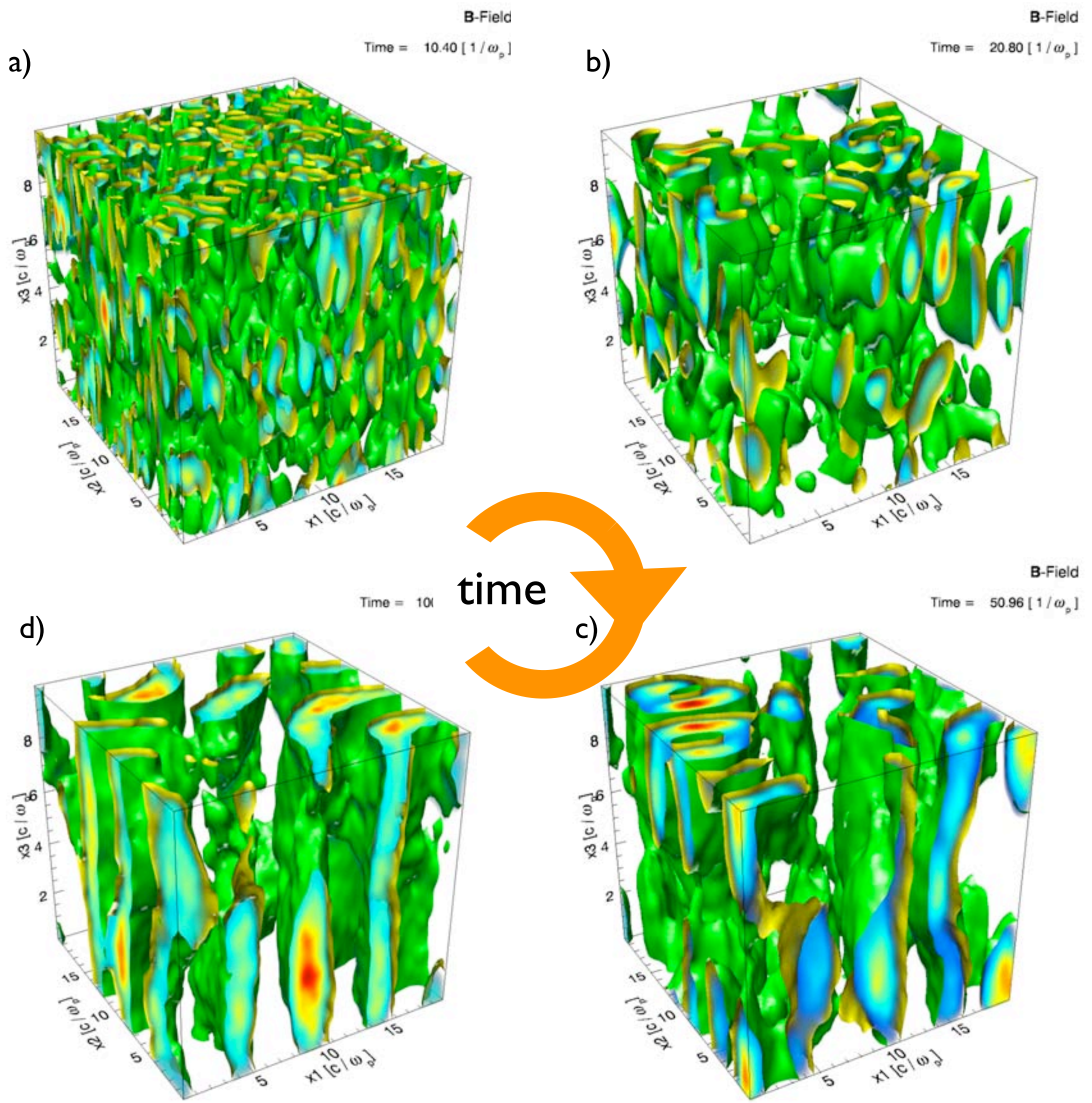
3D results - mass density



- Red Iso-surfaces: species with initial positive j_z
- Blue Iso-surfaces: species with initial negative j_z
- All isosurfaces drawn at a density value of 1.1 (1.0 being the initial density)
- a) t = 10.1 ω_p^{-1}, b) t = 20.8 ω_p^{-1}, c) t = 50.96 ω_p^{-1}, d)t =100.88 ω_p^{-1}

Species with opposite initial current are separated by regions of intense magnetic field.

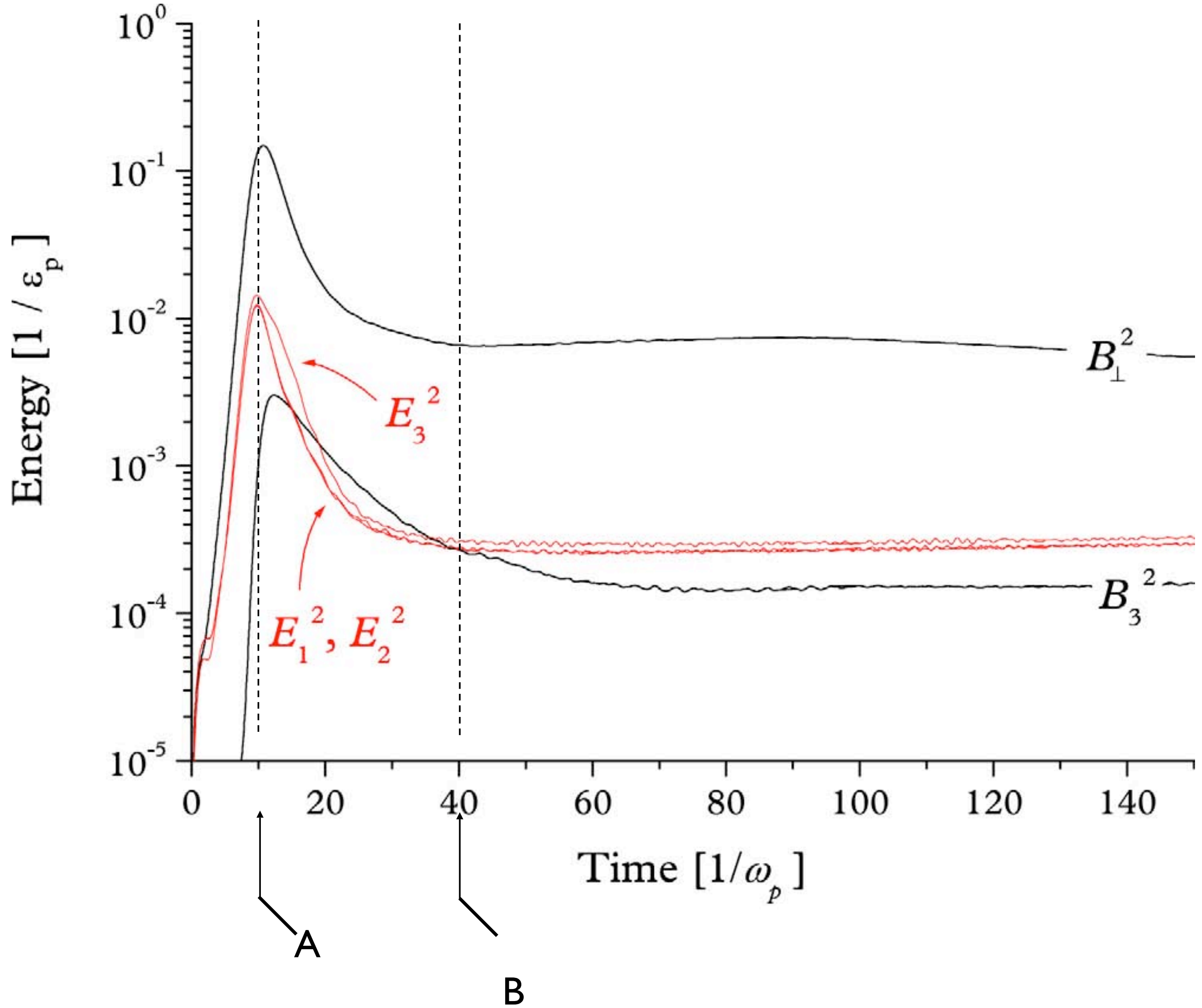
3D results - B-field generation



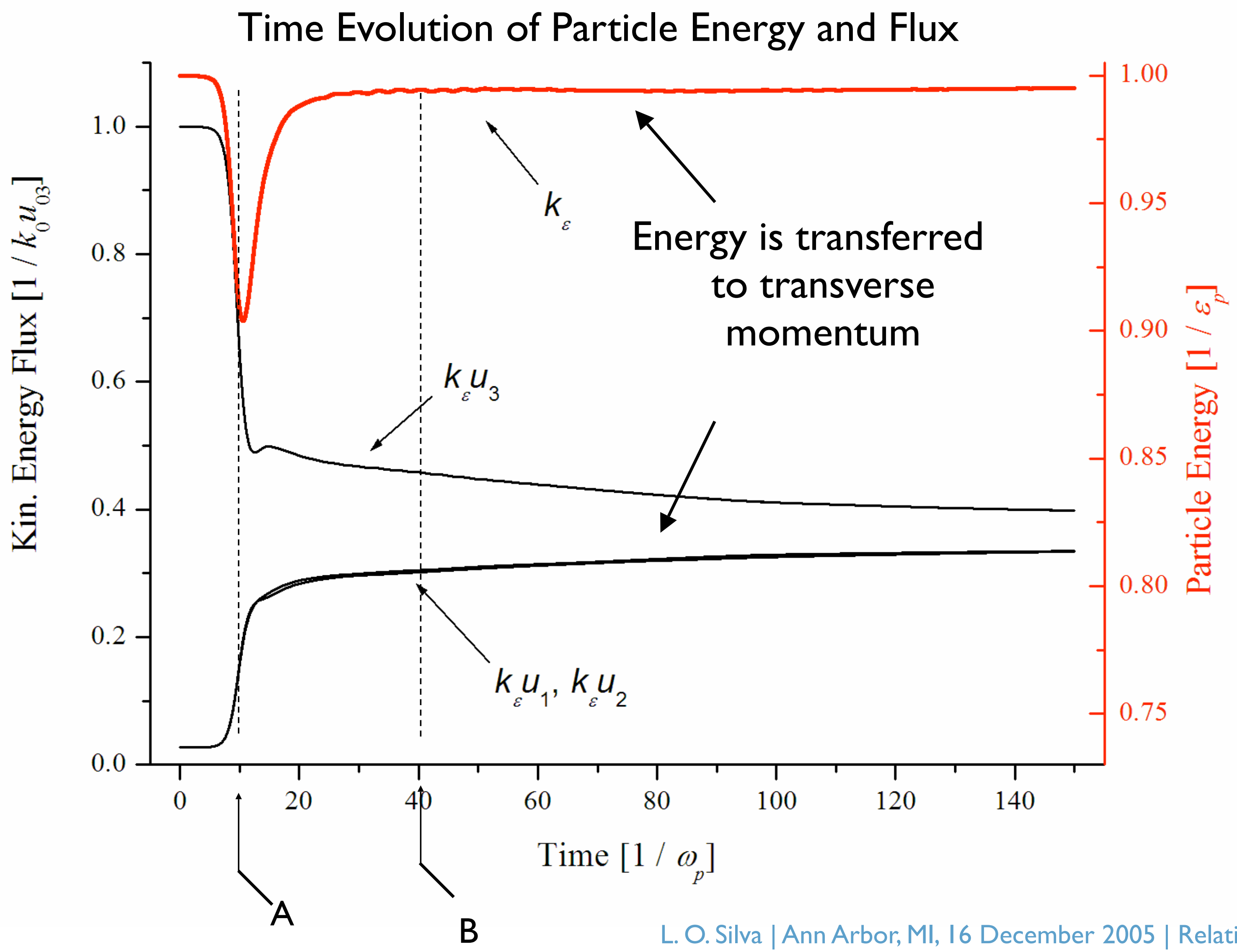
- Isosurfaces (Green - regions of lower values, Yellow regions of higher values) of the magnitude of the magnetic field
- Isosurfaces drawn at a) 0.1, b) 0.025, c) 0.01 and d) 0.006
- a) $t = 10.1\ \omega_p^{-1}$, b) $t = 20.8\ \omega_p^{-1}$, c) $t = 50.96\ \omega_p^{-1}$, d) $t = 100.88\ \omega_p^{-1}$

The self generated magnetic field exists in the region that separates species with initial opposite current

Field energy evolution - 3D Simulations

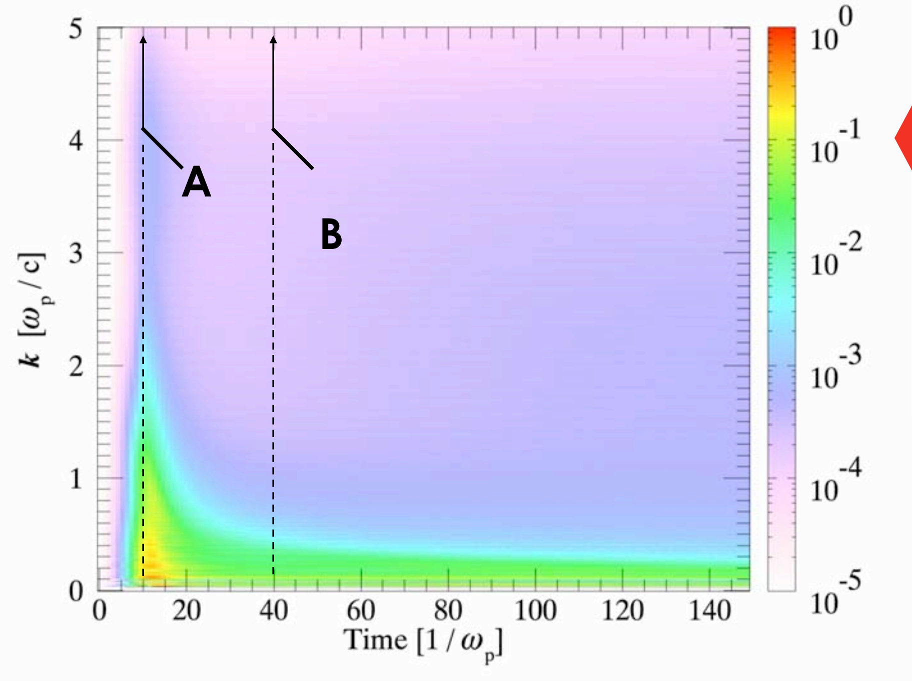


Particle energy evolution - 3D Simulations



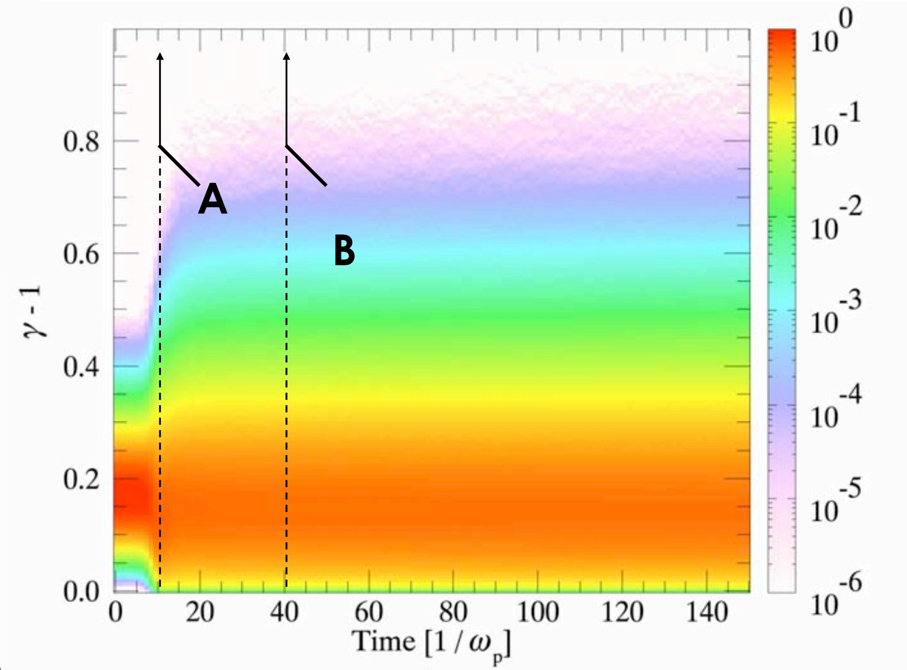
Field structure and particle energy distribution

B-Field Spectral Density Evolution



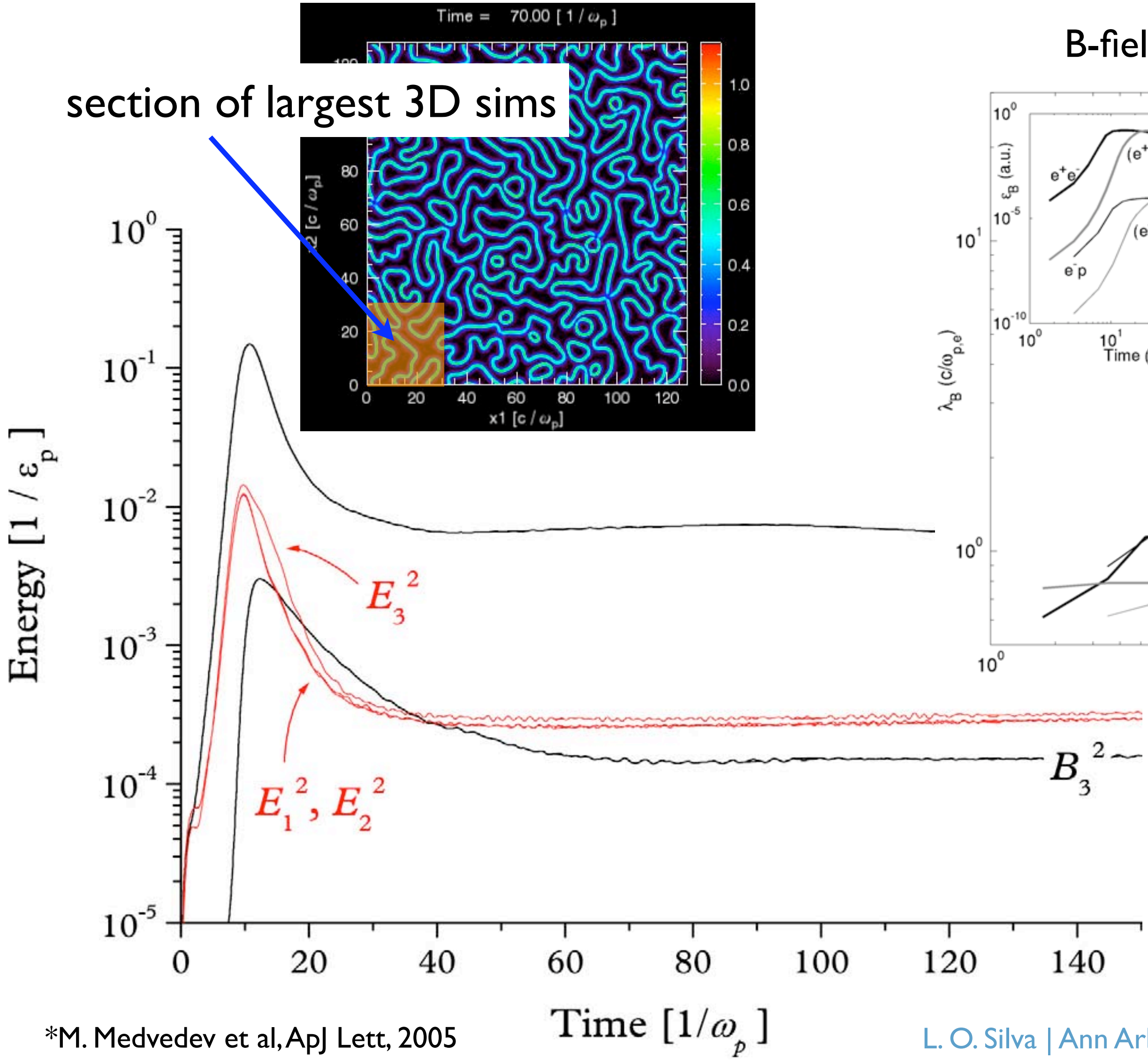
Magnetic field evolves from small wavelengths to large wavelengths (exponential decay)

Part. Energy Distribution Evolution

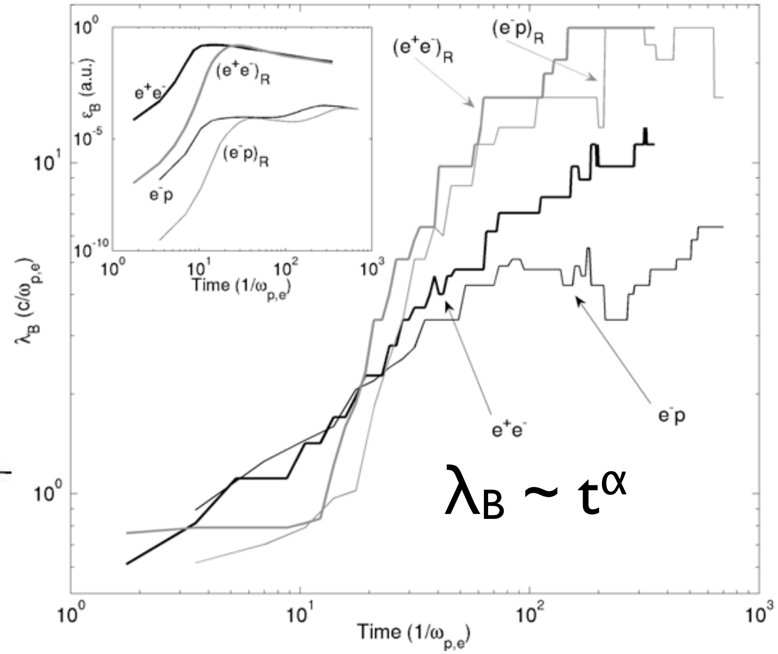


Strong transverse temperature increase and particle acceleration

Long time evolution of field structure*



B-field correlation length



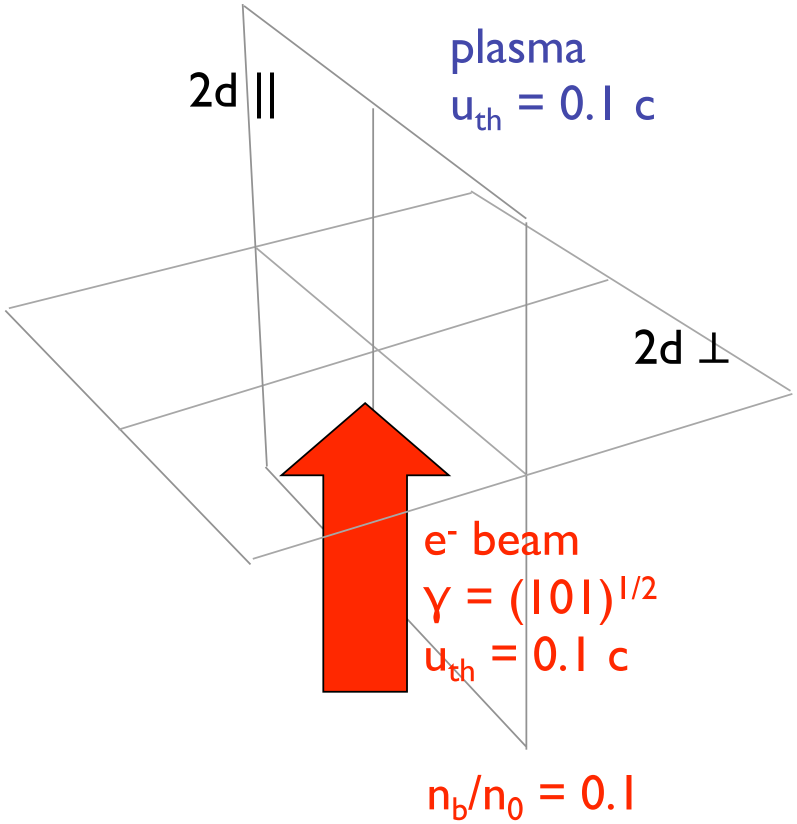
*M. Medvedev et al, ApJ Lett, 2005

What are the open questions/problems?

- Long time evolution of B-field for more realistic geometries
- Configuration of simulations not appropriate to check particle acceleration and relativistic shock formation:
  - either periodic systems or systems where plasma injected from side wall
- Collision of electron-"proton" shells
  - typical length scale and time scale increase by $(m_i/m_e)^{1/2}$
  - grand challenge for 3D simulations

Increase simulation box size and interaction time by reducing dimensionality of the simulations and using moving window

Simulation parameters (PARSEC)*



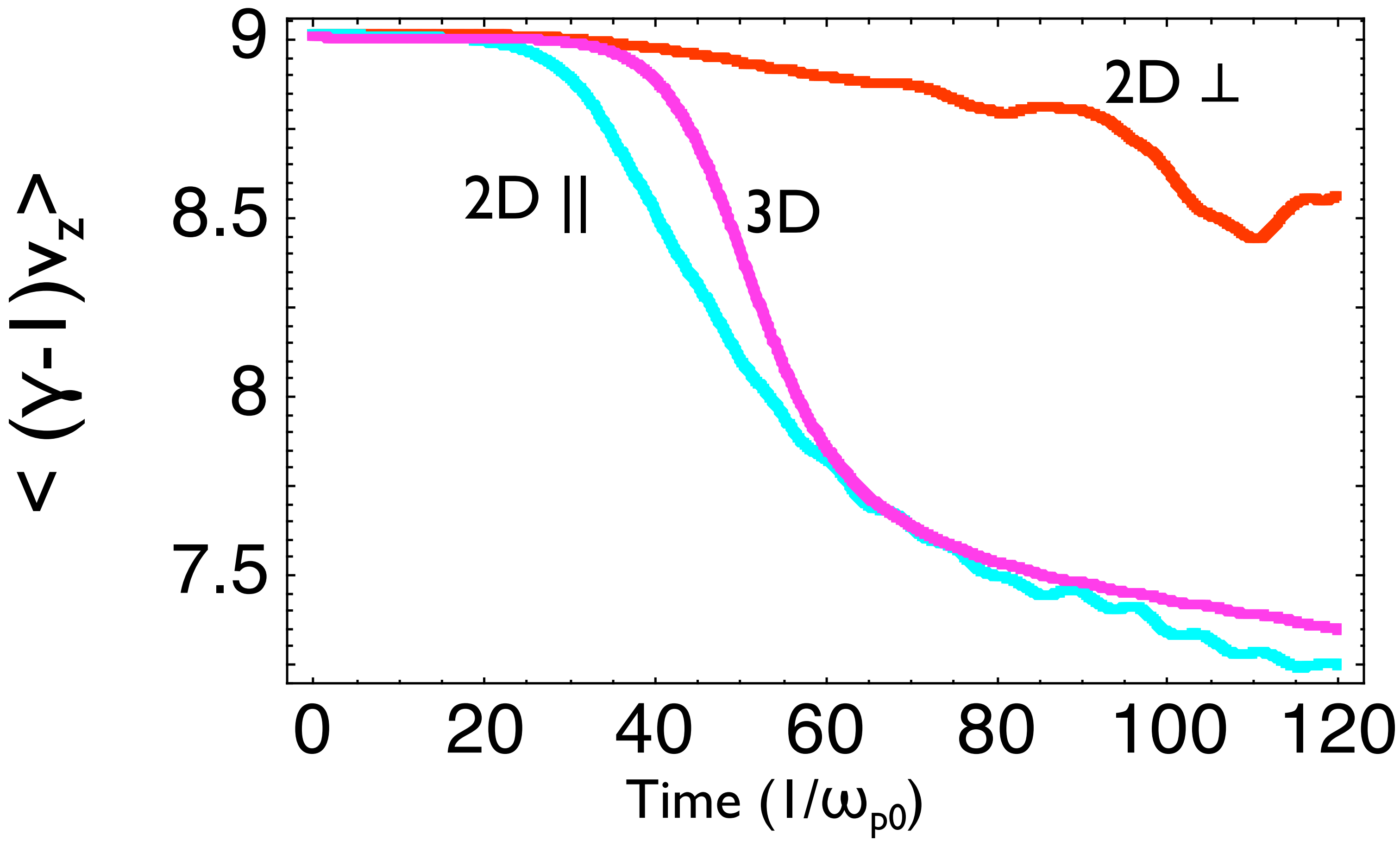
Periodic system
Current and charge neutral
Moving ions

- 3D runs
  - $(128)^3$ cells
  - $(12.8\ c/\omega_{p0})^3$
  - $\omega_{p0}t_{max}$=120
  - 16 particles/cell

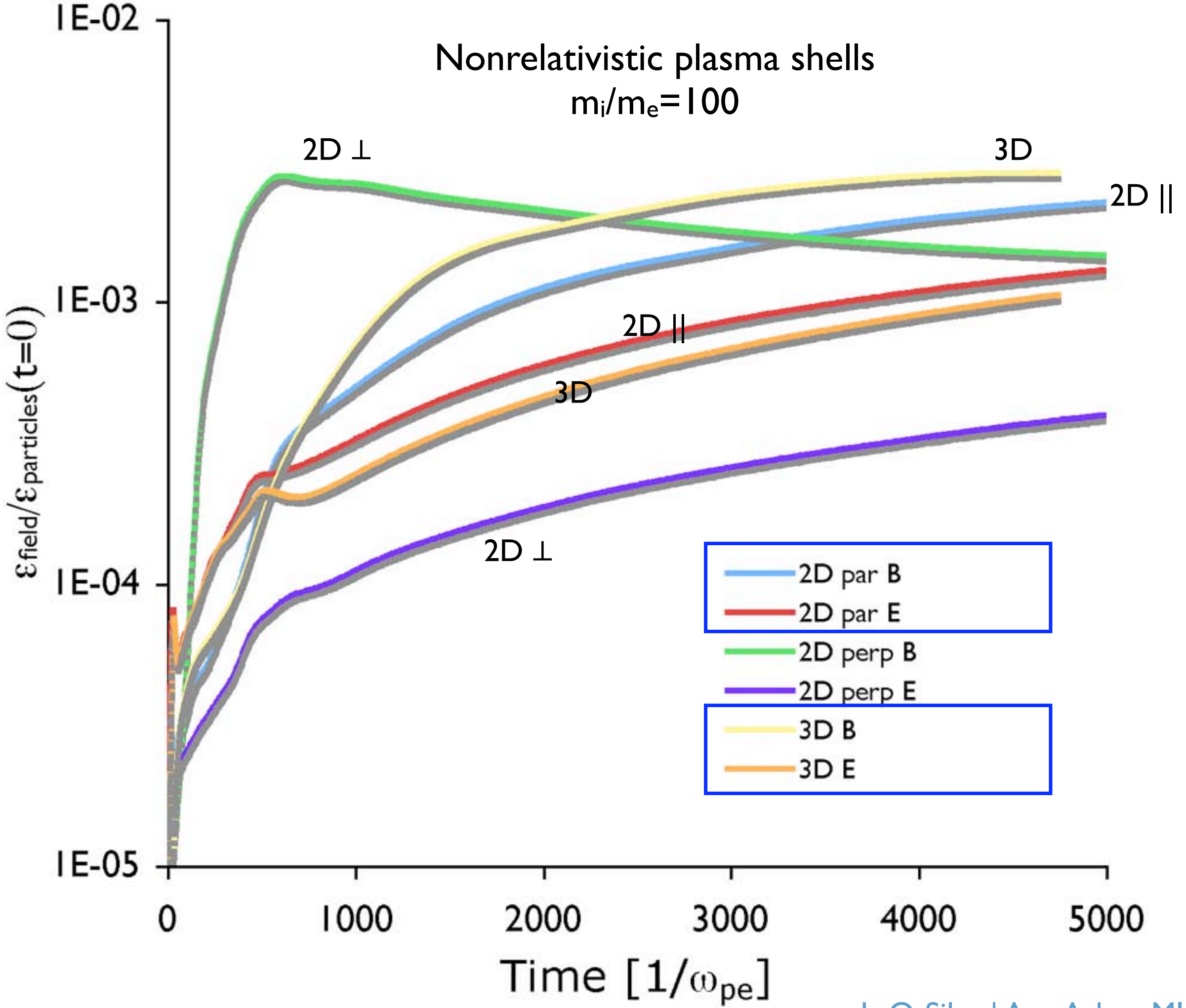
- 2D runs (|| and ⊥)
  - $(128)^2$ cells
  - $(12.8\ c/\omega_{p0})^2$
  - $\omega_{p0}t_{max}$=120
  - 16 particles/cell

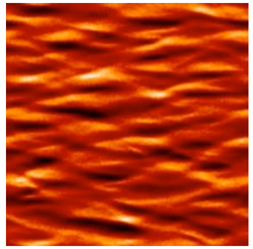
*J. Tonge, UCLA PhD Thesis, 2002

Kinetic energy flux per particle



Can lower dimensional simulations capture the [relevant] physics?



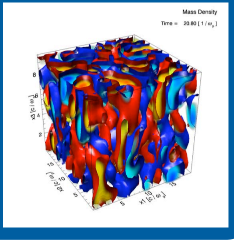


2D ||

2D ⊥

3D simulation box
$(25.6\ c/\omega_{p0})^3$
2D simulation box
$(256.0\ c/\omega_{p0})^2$

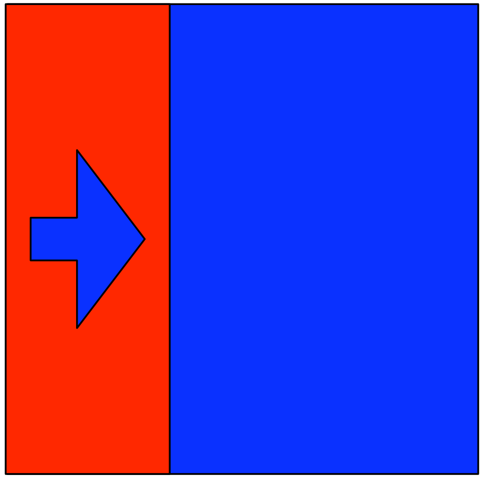
Magnetic field generation



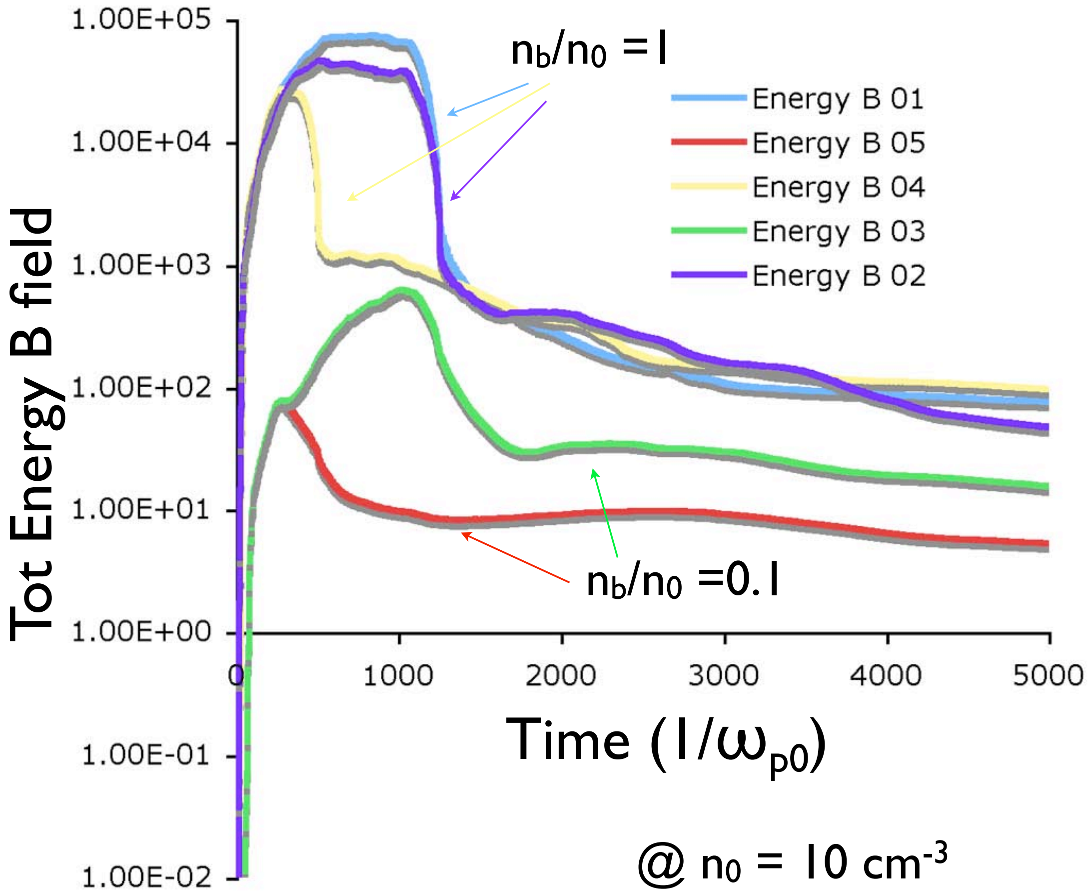
B-field evolution on large scales/long times

Relativistic (Υ = 10, 5) plasma shell injected from 2D simulation wall

m_i/m_e =100
16 particles/(cell species)



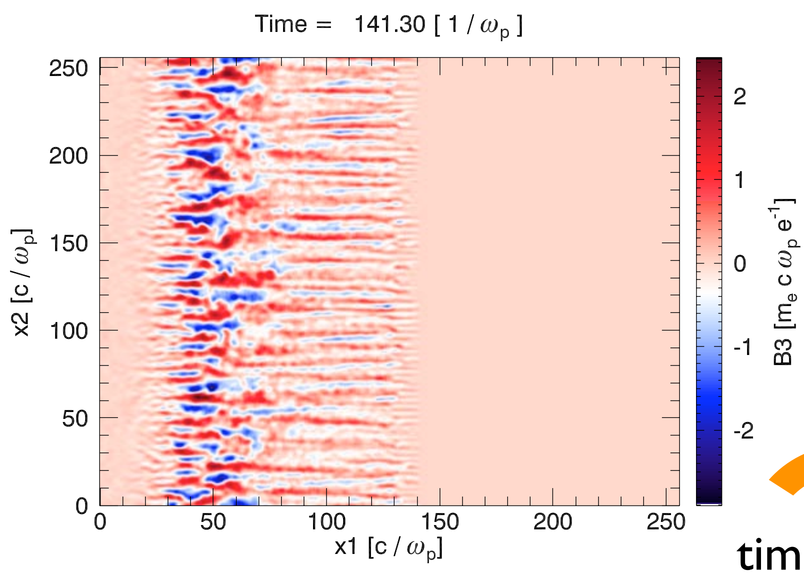
256 c/ω_{p0} × 256 c/ω_{p0}

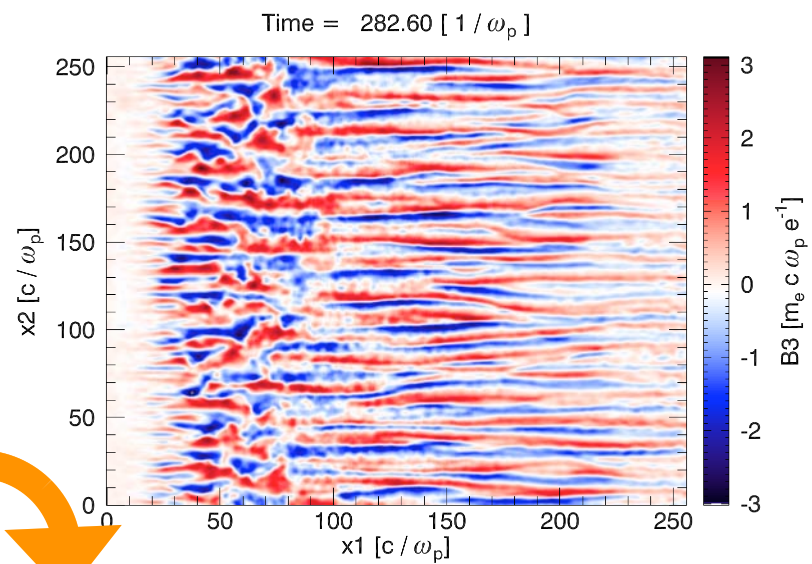


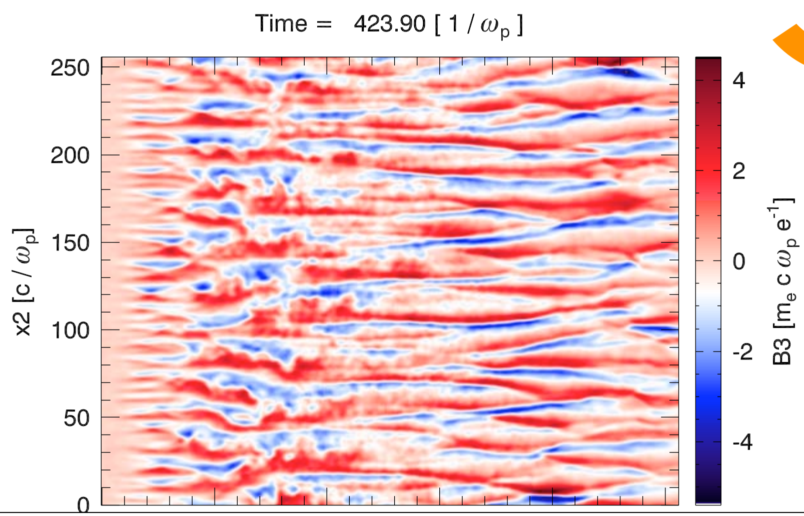
t_{on} = 0
t_{off} = $1000/\omega_{p0}$, $256/\omega_{p0}$
n_b/n_0 =1, 0.1

@ n_0 = 10 cm^{-3}
256 c/ω_{p0} ~ 400 km
$1000/\omega_{p0}$ ~ 5 ms

Weibel instability on electrons - B field



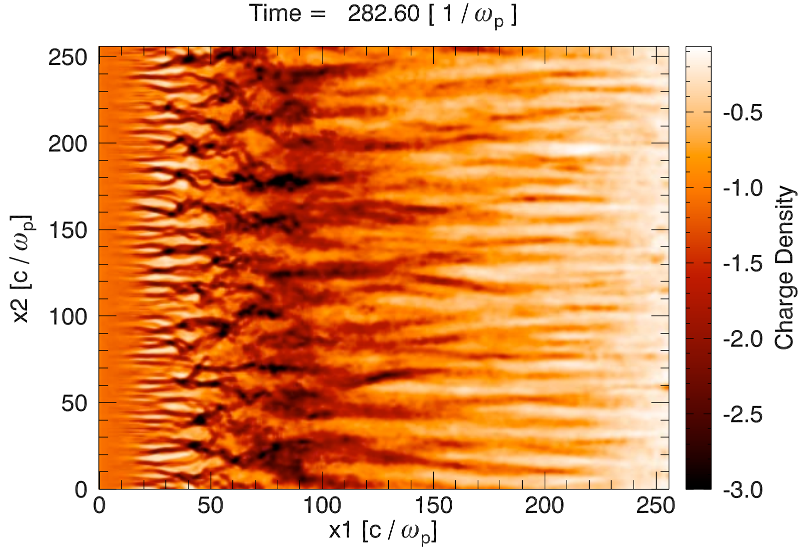


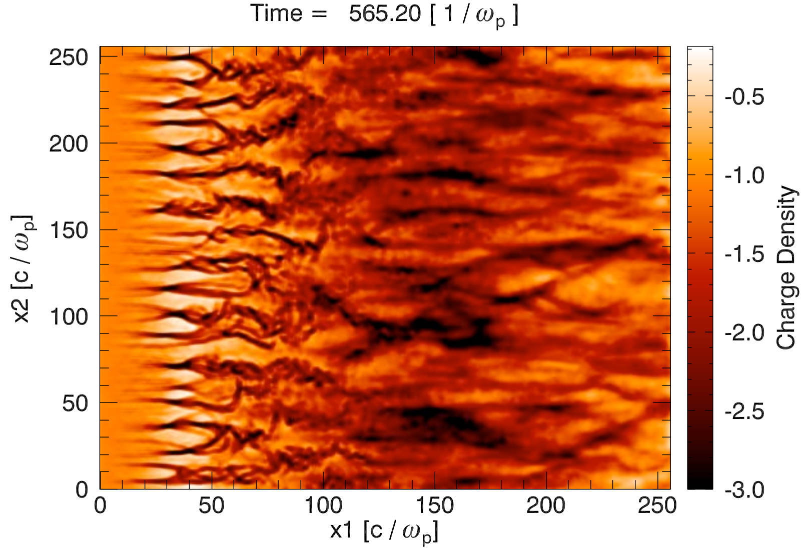


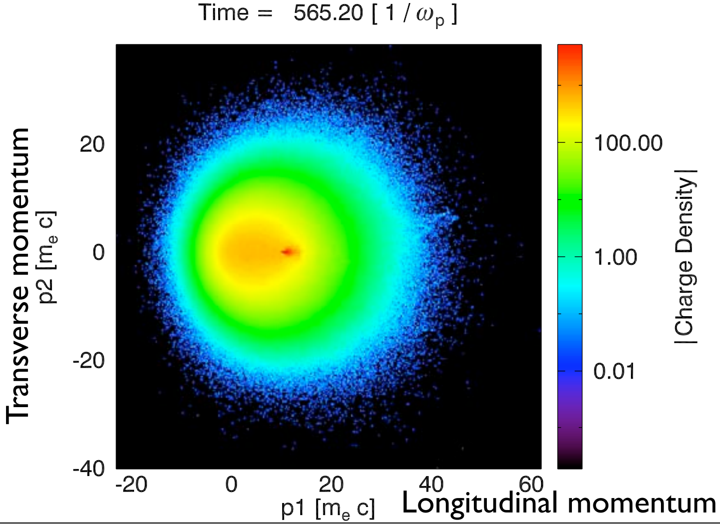
Spatio-temporal growth is evident with clear dynamics of tilted filament coalescence to large scales

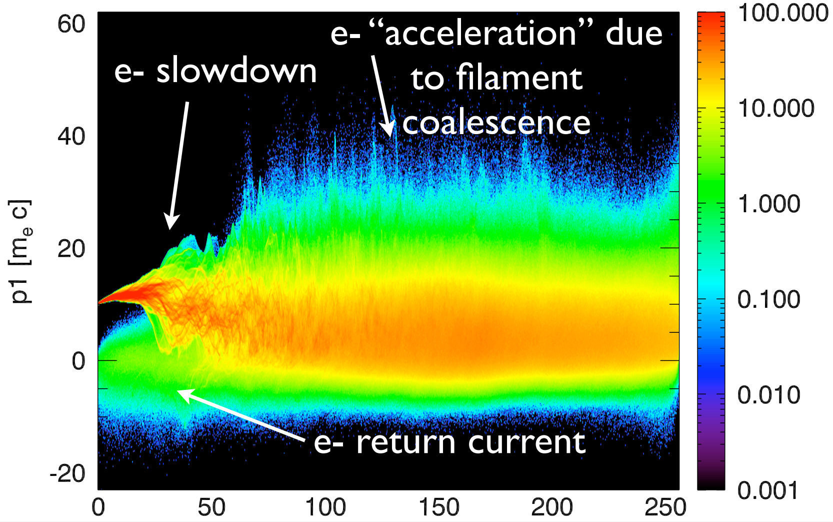
Spatio-temporal dynamics seeds Weibel on ions

Weibel instability on electrons

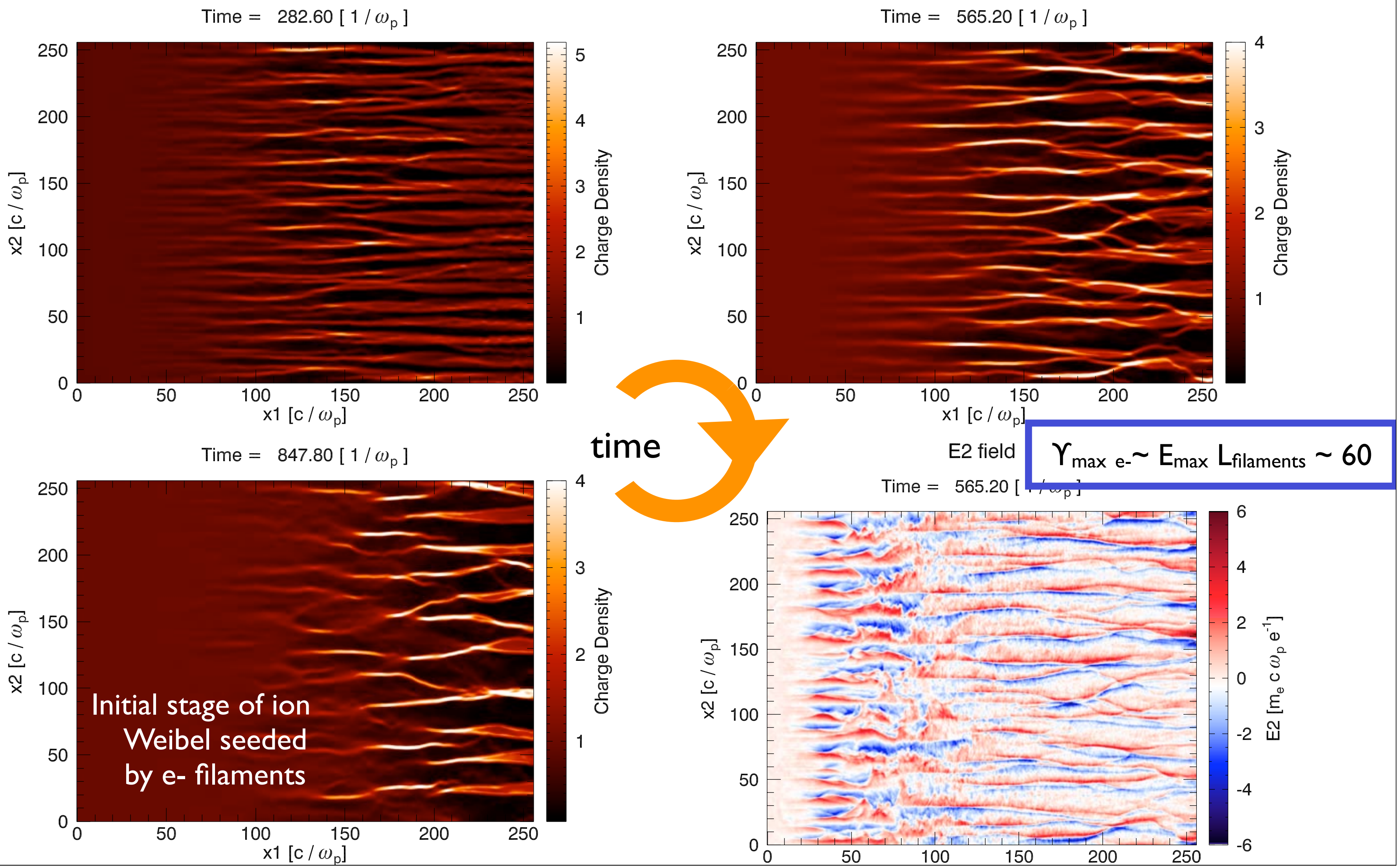




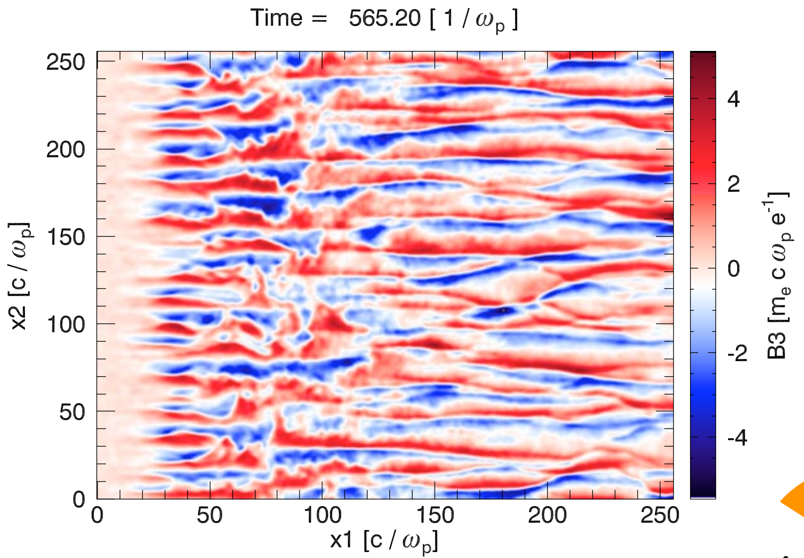


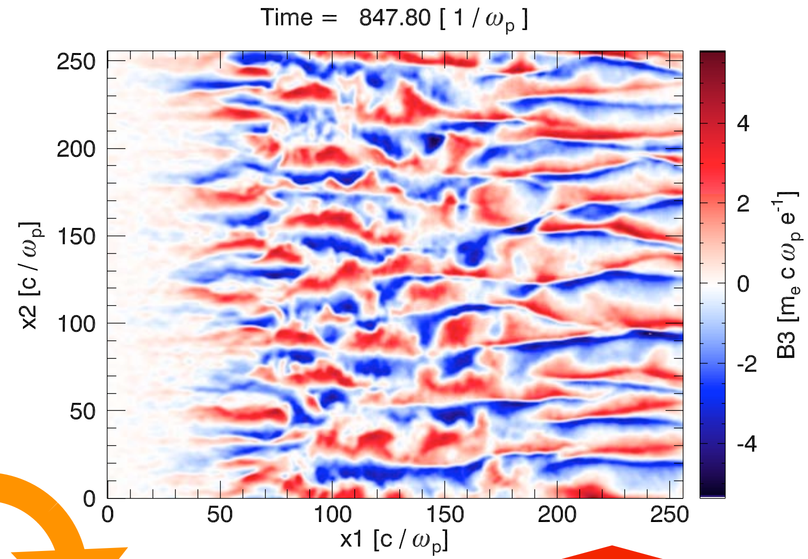


Weibel instability - ion density

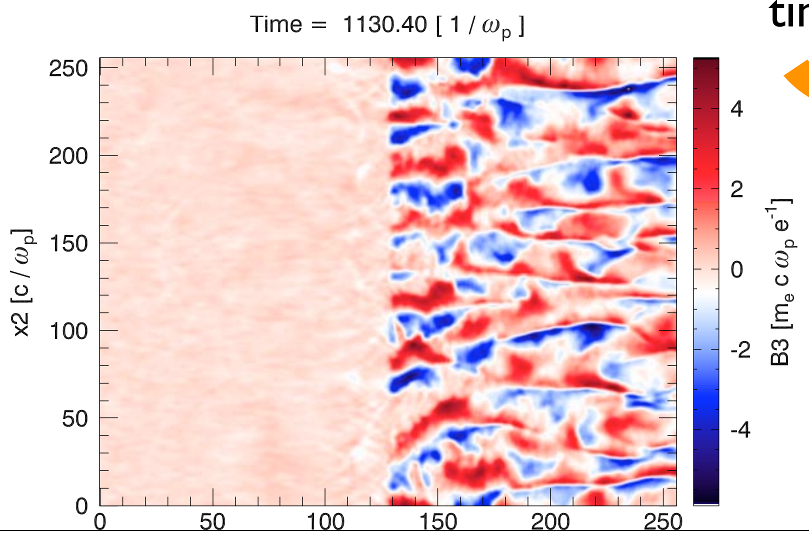


Weibel instability on ions - B field





time



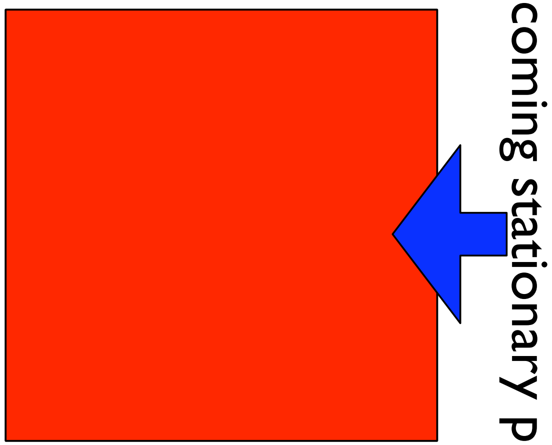
Large scale evolution can only be captured with long and wide box

For $n_b \sim n_0$ the self-generated field is transported by the relativistic plasma slab

B-field evolution on the head of relativistic shell I

$m_i/m_e = 100$

16 particles/(cell species)

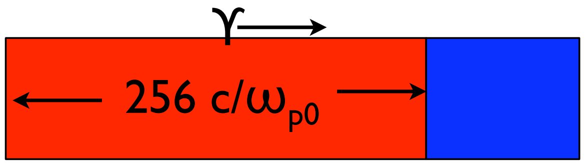


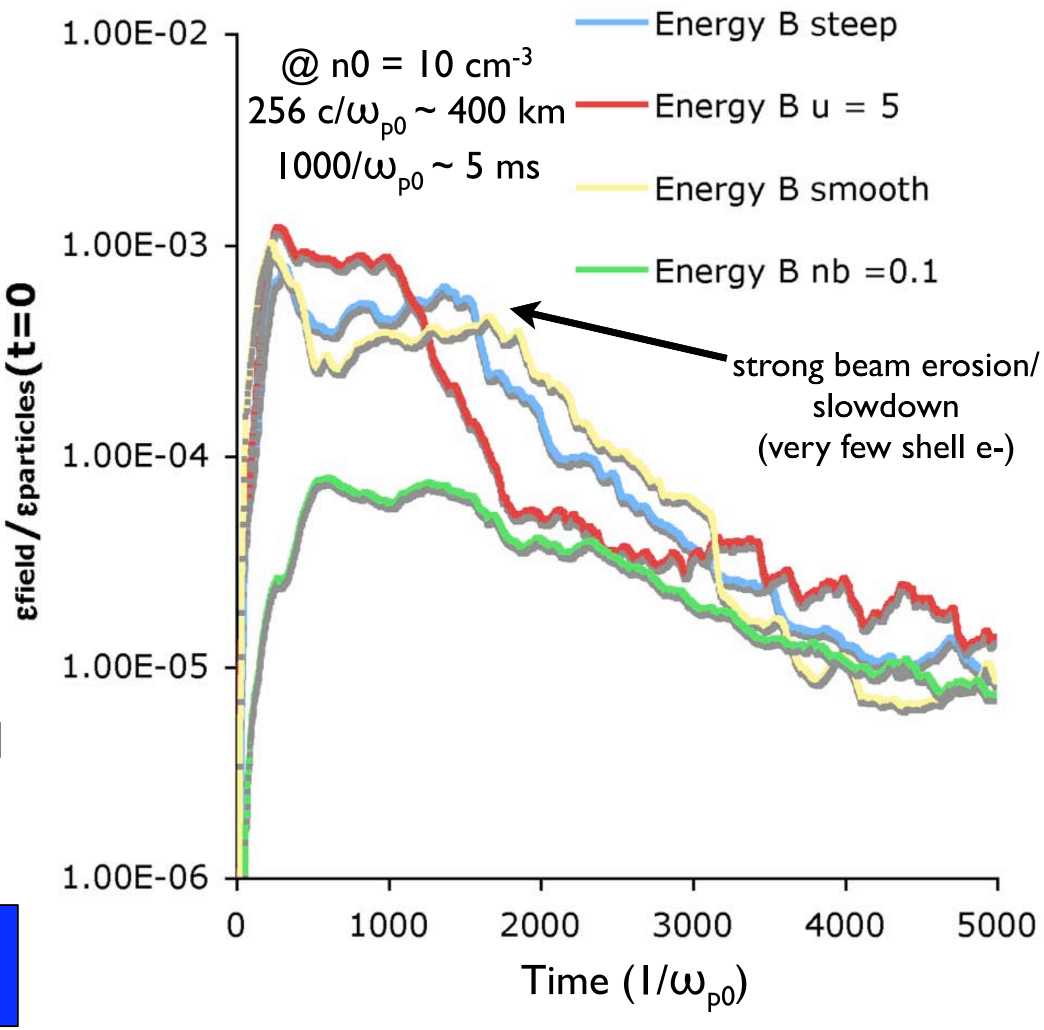
$256\ c/\omega_{p0} \times 256\ c/\omega_{p0}$

Moving window @ c

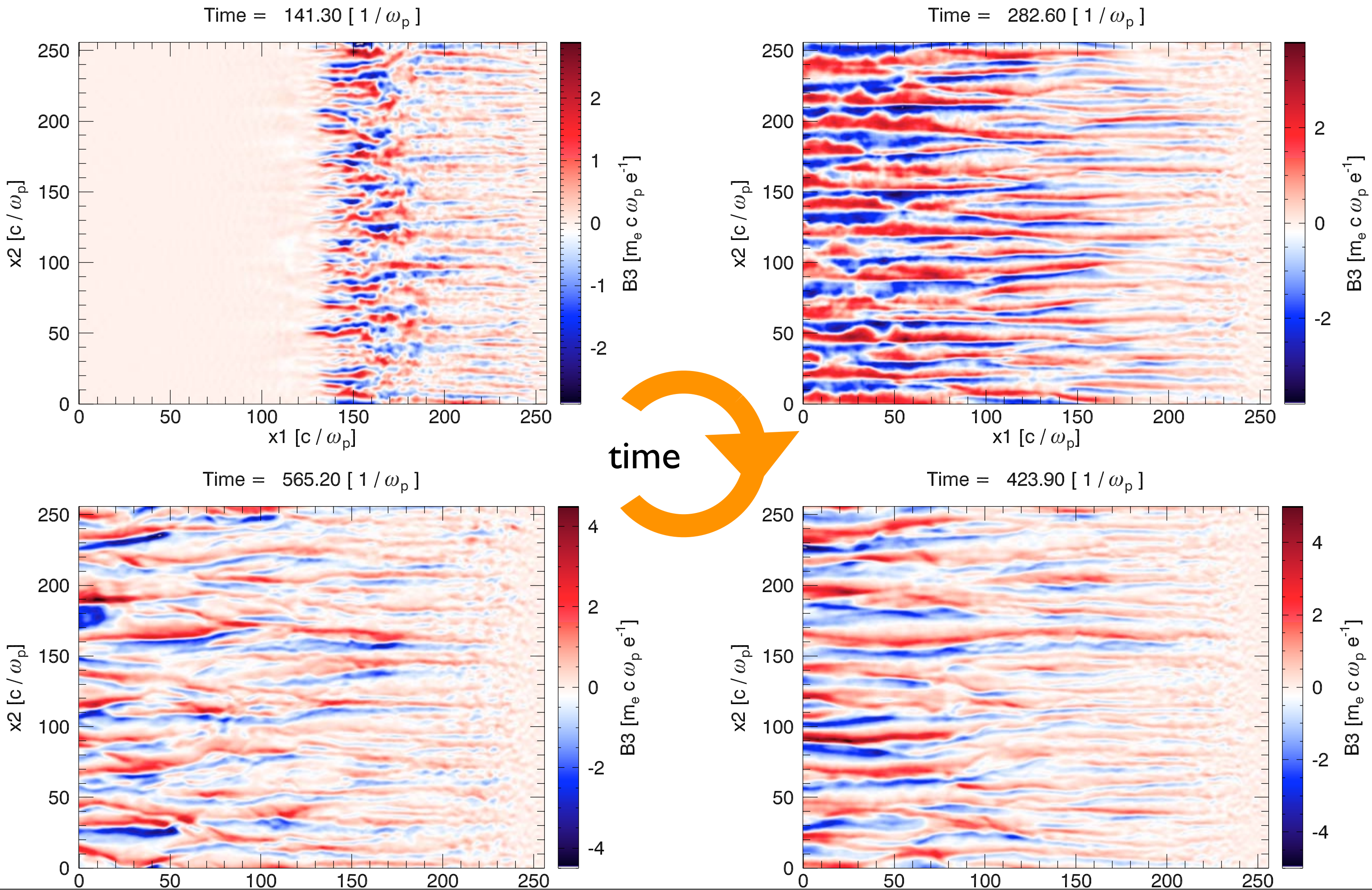
Relativistic (Υ = 10, 5) plasma shell

$n_b/n_0 = 1, 0.1$

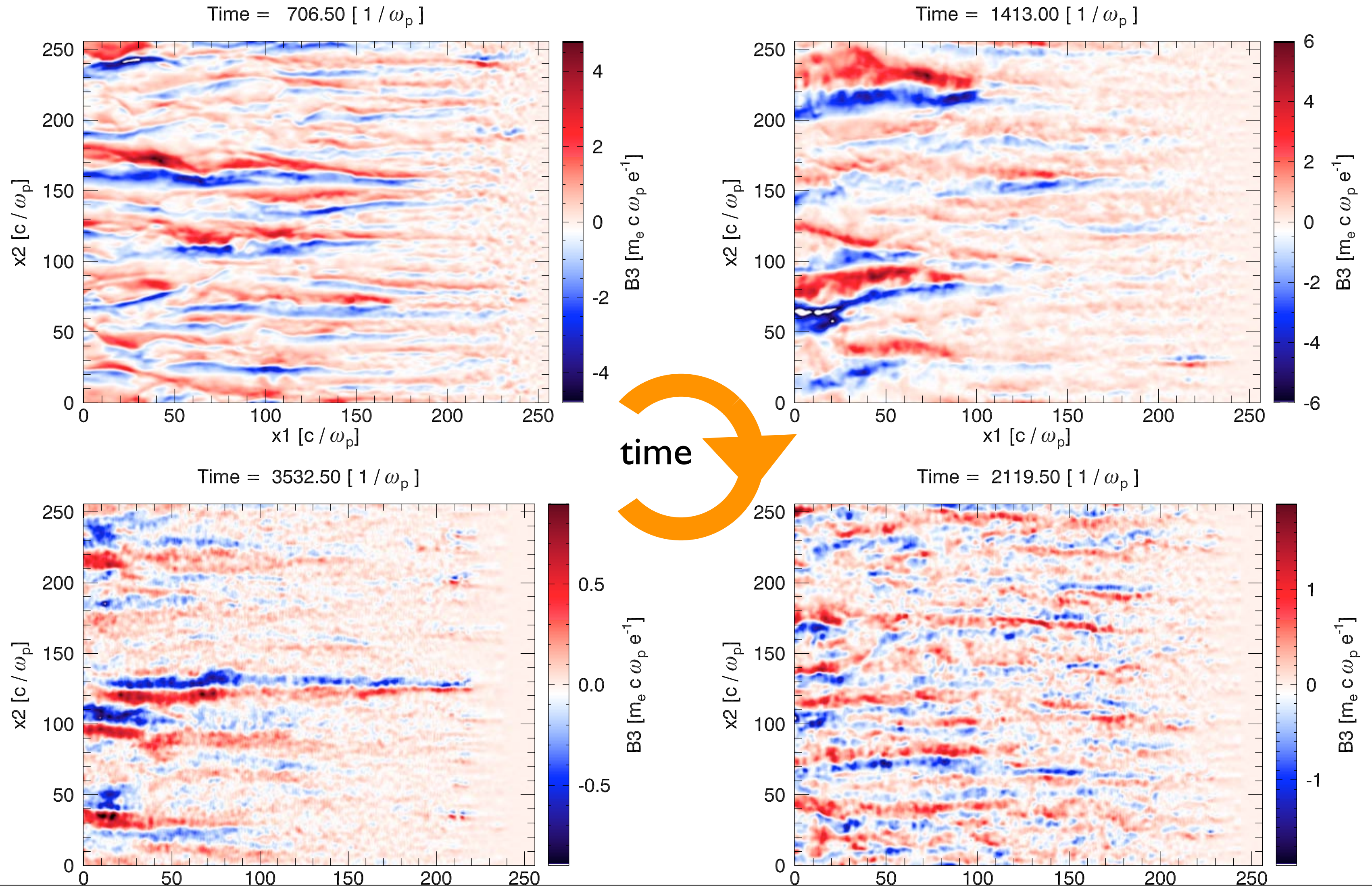




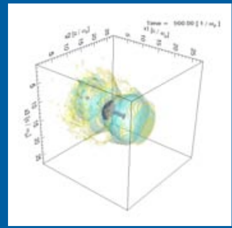
B-field on the head of relativistic shell - Weibel on e-



B-field on the head of relativistic shell - Weibel on i

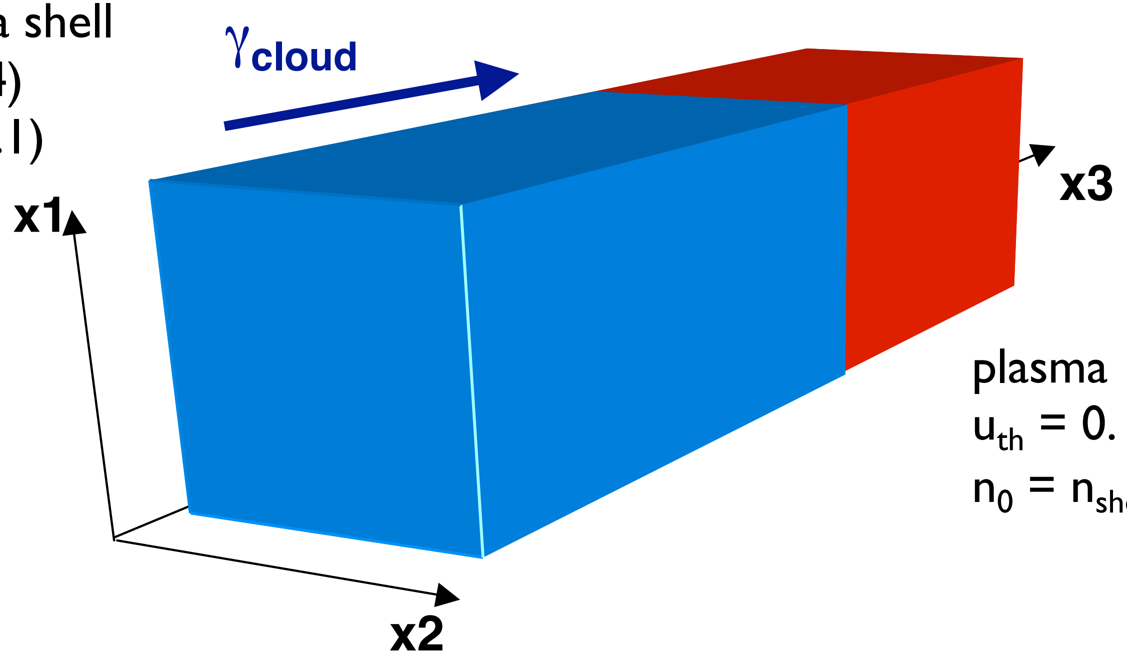


Particle acceleration in plasma collisions



3D PIC simulations of e-e+ shell collisions with moving window

e^+e^- plasma shell
γ = 10 (1.4)
u_{th}/c= 1 (0.1)

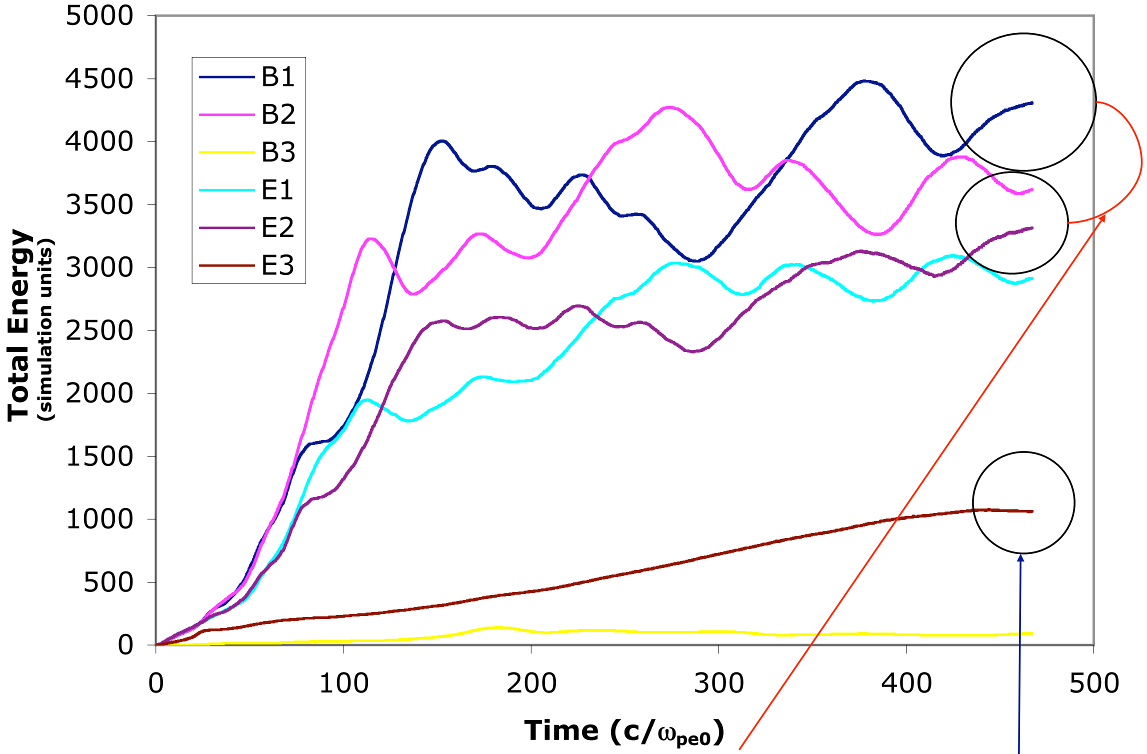


plasma
u_{th} = 0.1 c
$n_0 = n_{shell}$

3D runs

64×64×1600 cells
12.8×12.8×320.0 $(c/\omega_{p0})^3$
$\omega_{p0}t_{max}$= 500 (250)
8 particles/(cloud×cell)
Moving window (@ c) follows shock front

Time history for fields (γ_{shell} = 10)



Quasi-steady state in the fields

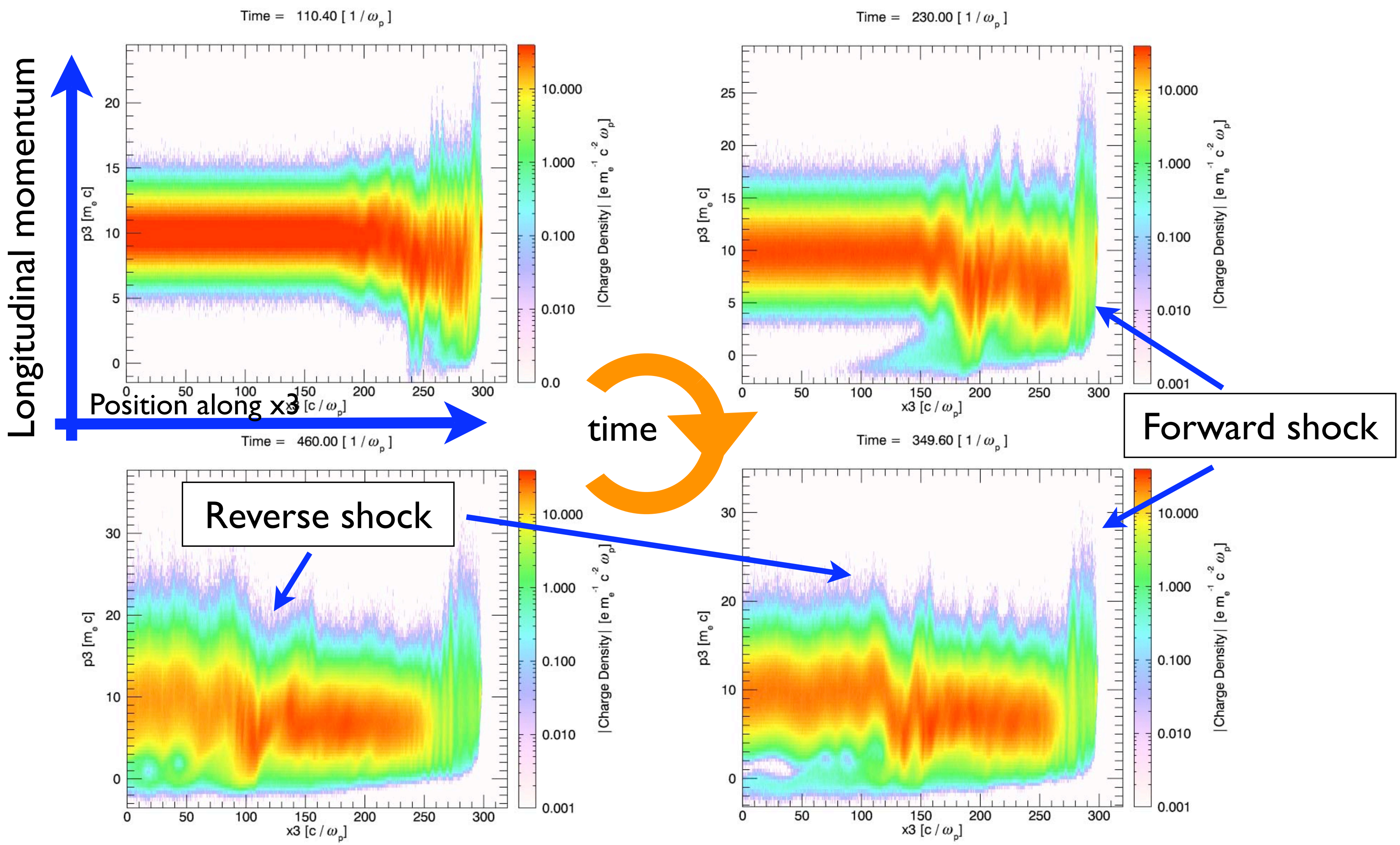
Total energy in B-field ~ 10^{-4} total initial kinetic energy

Comparable energy in B and E (crucial for particle acceleration)

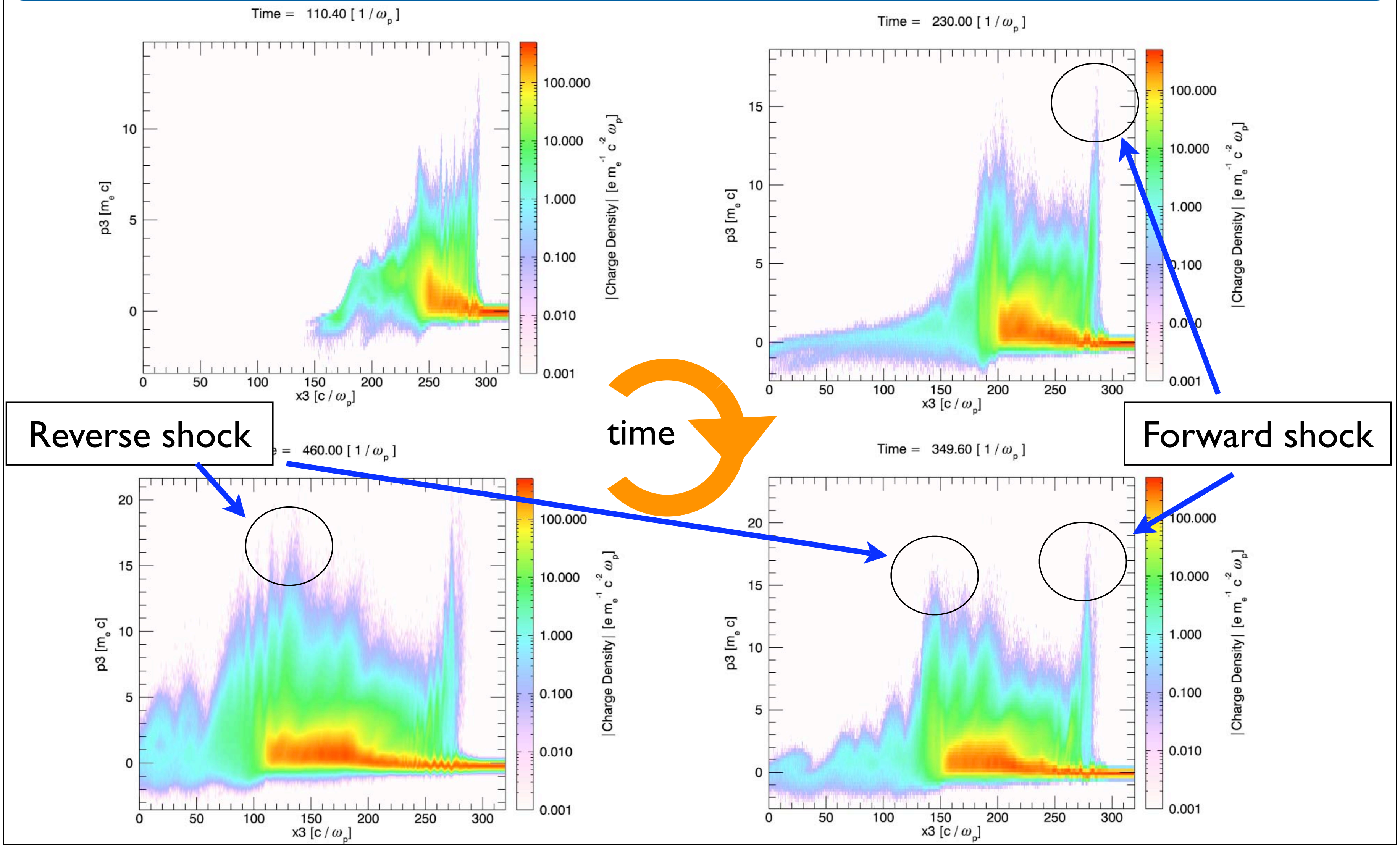
E_1 and E_2 comparable to B_1 and B_2 (e.m. field)

E_3 is also significant (crucial for particle acceleration)

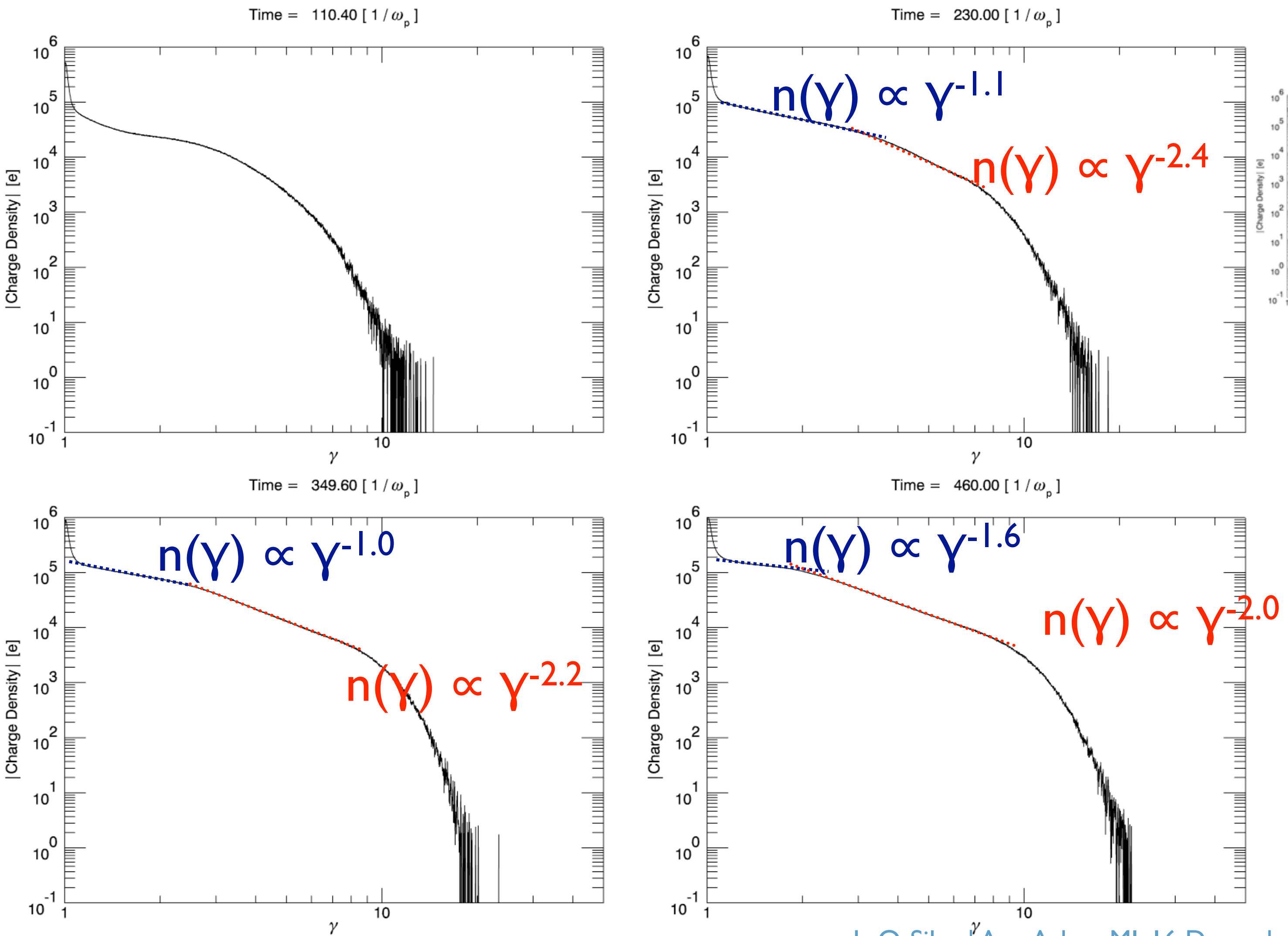
Shock formation (γ_{cloud} = 10) - shell particles

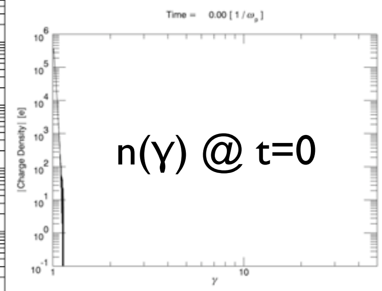


Shock formation (γ_{shell} = 10) - background plasma



Particle acceleration (γ_{shell} = 10) - background plasma





Power Law

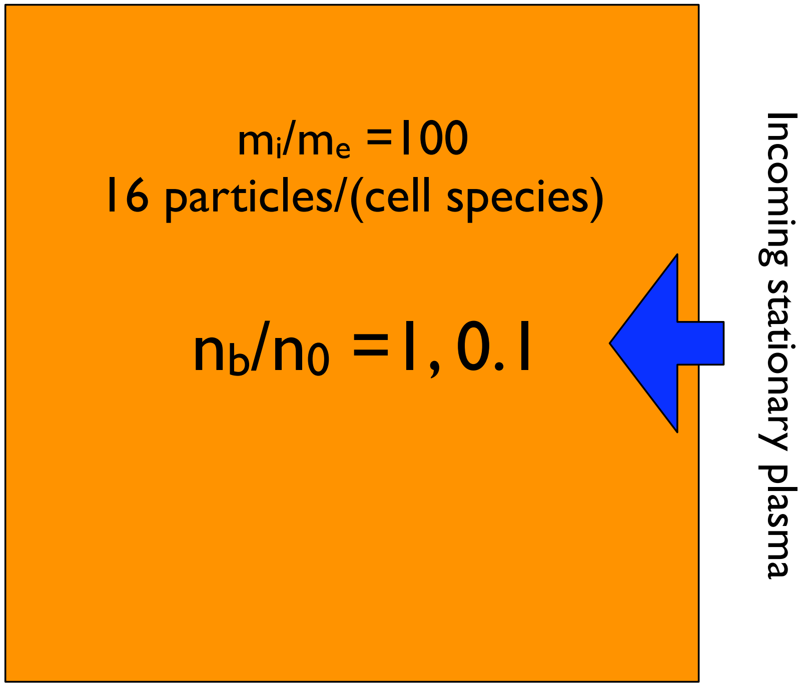
$n(\gamma) \propto \gamma^{-\alpha}$

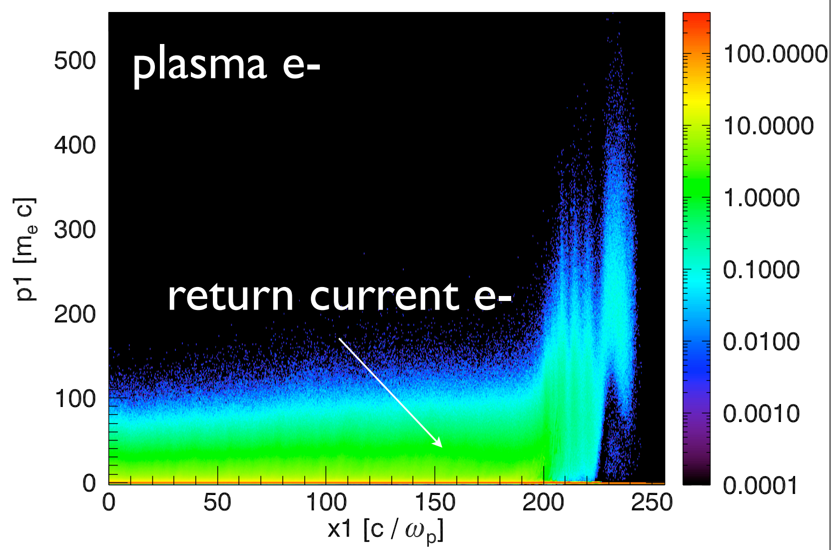
2D PIC e$^-$ i shells and particle acceleration (γ_{shell} = 10)

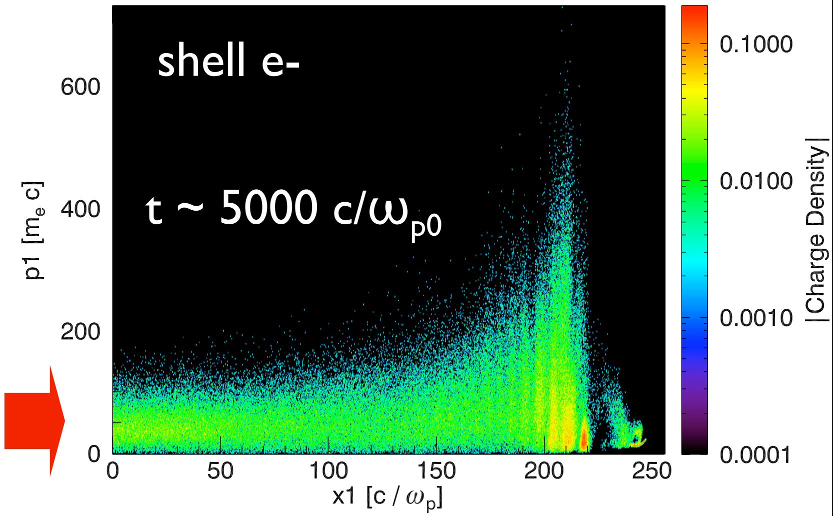
Moving window @ c

Relativistic (Υ = 10, 5) plasma shell

256 c/ω_{p0} × 256 c/ω_{p0}



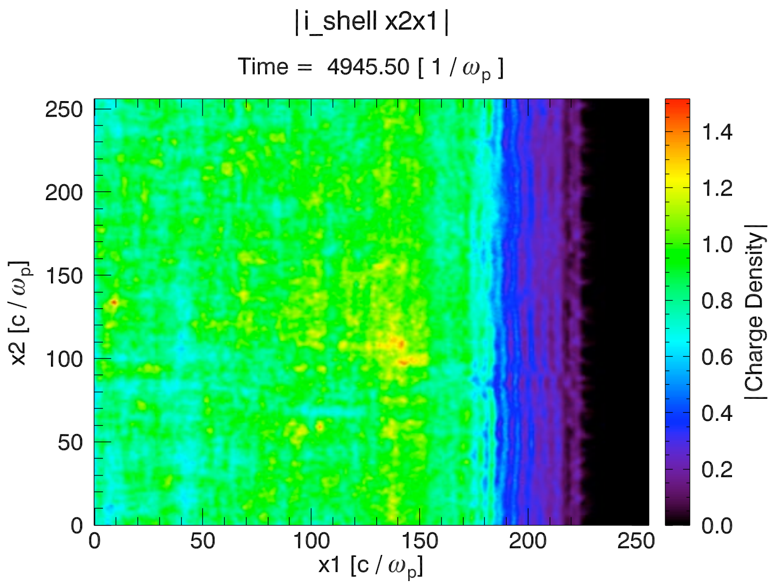




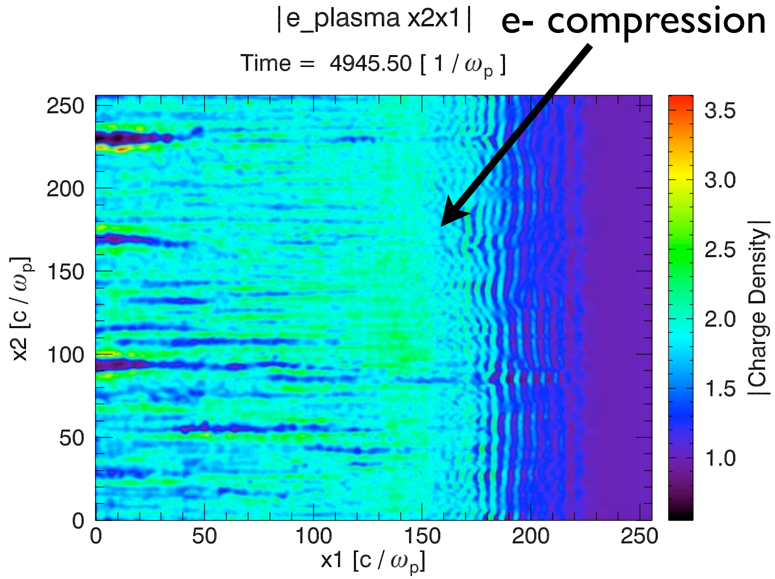
Strong erosion of shell e- @ later times

(Relativistic) Buneman responsible for accelerating structure

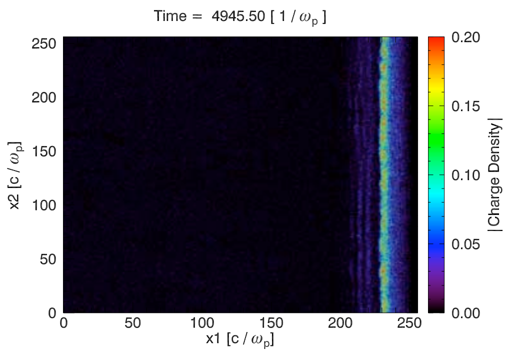
Strong electric field due to Buneman instability of ion shell with ambient background plasma



e- compression region



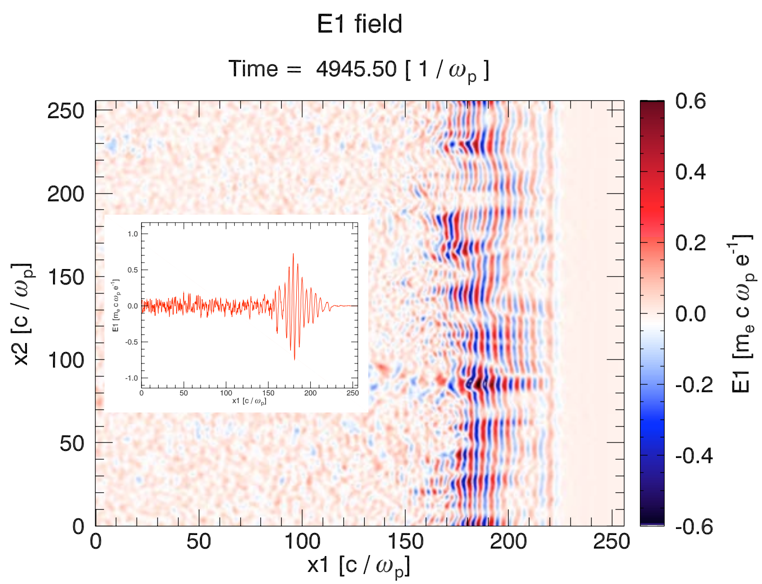
Small fraction of trapped electrons from the shell

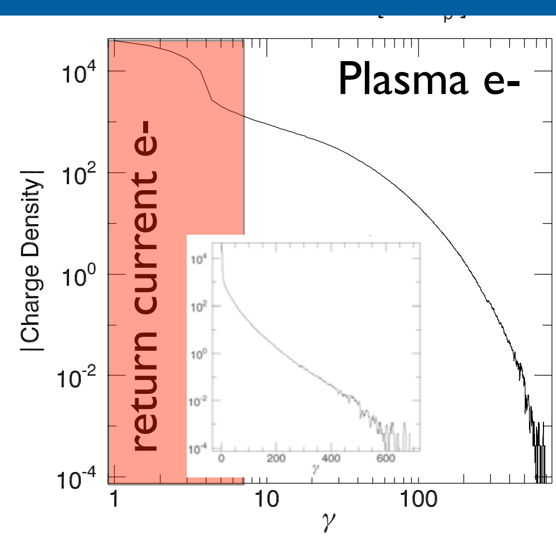


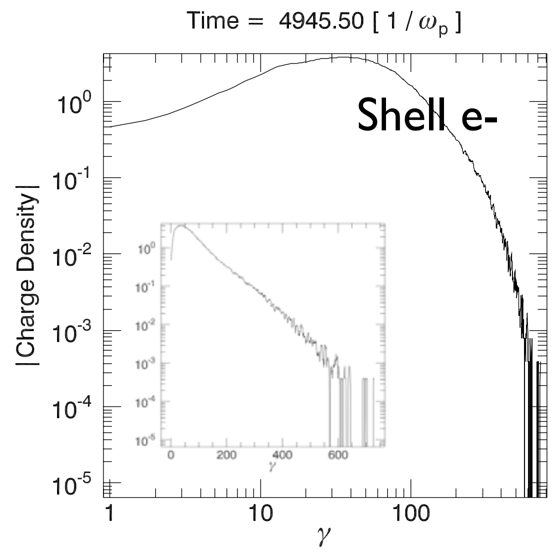
$$k_{||} \approx \omega_{p0}/c$$

$$\Gamma \propto \sqrt{\frac{n_b}{n_0}}\sqrt{\frac{m_e}{m_i}}$$

Strong acceleration by parallel E field in leading edge of shell







Accelerating structure moving with $\gamma_\phi \sim \gamma_{shell} \sim 10$

$$\gamma_{\rm max} \approx 4\,\gamma_\phi^2\,\tilde{E}_{\rm max}$$

$\tilde{E}_{\rm max} \approx 1 - 2$ (as measured in simulations, waves close to wavebreaking)

$$\tilde{E}_{\rm max} = E_{\rm max}/E_0 \approx E_{\rm max}[V/cm]/\sqrt{n_0[\rm cm^{-}3]}$$

Summary

- Particle-in-cell simulations as a tool to probe the microphysics of relativistic flows
- 3D simulations are very expensive but lower dimensional simulations can capture some of the physics
- Very large 2D simulations seem to confirm sub-equipartition B-field generation for very long times with ions ($\epsilon_B \sim 10^{-3}$, up to $2000/\omega_{p0}$) - in progress largest 2D simulations (4 x size)
- Dynamics of leading edge of relativistic shell
  - strong erosion of e- in shell
  - formation of hot return current with plasma e-
  - onset of Buneman instability leads to e- acceleration up to $\sim \Upsilon_{shell}^2$