Math 471-001 Fall 2013 Homework 1 due : Thursday, Sept. 12

0. Give a brief description of your academic background and research interests. If you work in a lab, please give your supervisor's name and describe your project. One paragraph is fine.

floating point representations

1. (a) Convert $(1775)_{10}$ to binary. (b) Convert $(100111.01)_2$ to decimal.

2. A floating point representation of a real number has the form $\pm(0.d_1 d_2 \dots d_n)_\beta \cdot \beta^e$ where $d_1 \neq 0$ and $-M \leq e \leq M$. Consider a system with $\beta = 2, n = 4$, and $M = 5$.

   (a) Find the largest and smallest positive numbers than can be represented by this system. Give your answers in decimal form.

   (b) Find the floating point number in this system that is closest to $\sqrt{2}$.

3. The polynomial $p(x) = (x-2)^9$ clearly has a root at $x = 2$.

   (a) Matlab. Plot $p(x)$ as given above and in its expanded form

   $$p(x) = x^9 - 18x^8 + 144x^7 - 672x^6 + 2016x^5 - 4032x^4 + 5376x^3 - 4608x^2 + 2304x - 512$$

   on the same set of axes. Use `x=1.92:0.001:2.08;` and be sure your axes are legible and the two curves are clearly distinguished. (Hint : Change the axes labels' font size with the command `set(gca,'FontSize',18)`, and look up `LineSpec` and `legend` in the Matlab documentation.)

   (b) What accounts for the differences you see in part (a)?

4. page 40, problem 8(a) (Read "An example to set the stage," on page 31.)

5. page 52, problem 13. In part (a), assume that $1 + \sin x \neq 0$.

6. Consider the equation $x^2 + 25x + 0.1 = 0$.

   (a) Solve for the roots using the quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ and 4-digit (decimal) arithmetic, as in the example from class (and page 45). Compare with results obtained by using Matlab.

   (b) Repeat using the alternative form of the quadratic formula, $x = \dfrac{2c}{-b \pm \sqrt{b^2 - 4ac}}$.

   Compare the results with part (a) and explain your observations. What is different about this example compared to the one in the text?

finite differences

7. In class we defined the forward difference approximation, and the forward finite difference operator,

   $$D_+ f(x) = \frac{f(x+h) - f(x)}{h}$$

   for a given step size $h > 0$.

(a) Matlab. Take $f(x) = \cos x$, $x = \pi/4$, and $h = 2^{-n}$ for $n = 1, 2, \ldots, 6$. Following the example from class, plot error versus h and make a table with the following format: column 1 : h, column 2: $D + f(x)$, column 3: $f'(x) - D_+f(x)$, column 4: $(f'(x) - D_+f(x))/h$, column 5: $(f'(x) - D_+f(x))/h^2$. Present at least 8 decimal digits of precision (type `format long` in Matlab to get the full 15 digits).

(b) Matlab. Repeat part (a) for the centered finite difference operator

$$D_0f(x) = \frac{f(x+h) - f(x-h)}{2h},$$

which also approximates $f'(x)$.

(c) Which approximation is more accurate? Explain how you know, and why this is the case. Perhaps, you could even sketch a plot to guide your interpretation, similar to the one shown in class for $D_+f(x)$.