

Helmholz equation

1. Consider the two-dimensional eigenvalue problem for a general domain $D \subset \mathbb{R}^2$,

$$\nabla^2 \phi + \lambda \sigma(x, y) \phi = 0$$

where $\phi(x, y) = 0$ on ∂D and $\sigma(x, y) > 0$ for all (x, y) .

- (a) Prove that $\lambda \geq 0$.
- (b) Is $\lambda = 0$ an eigenvalue? If so, what is the corresponding eigenfunction?
- (c) Answer (a) and (b) if the boundary condition is $\nabla \phi \cdot \hat{\mathbf{n}} = 0$ on ∂D .
- (d) Answer (a) and (b) if the boundary condition is $\nabla \phi \cdot \hat{\mathbf{n}} + h(x, y) \phi = 0$ on ∂D .

nonhomogeneous problems

2. page 346, problem 8.2.2 a, b. Use either a reference solution or Green's formula. Note that you only need to find the related homogeneous problem (not the entire solution).
3. page 364, problem 8.5.2 b
4. page 364, problem 8.5.3
5. page 371, problem 8.6.1 d
6. page 372, problem 8.6.6

Fourier transforms

7. page 447, problem 10.3.5
8. page 447, problem 10.3.6
9. page 448, problem 10.3.8