Math 454-001 Winter 2014 Homework 8 due : Tuesday, April 22

Green's functions

1. In class we derived the Green's function for the 1D Poisson problem,

$$\frac{d^2u}{dx^2} = f(x), u(0) = 0 \quad u(L) = 0.$$

Using the method of eigenfunction expansion, we found that

$$G(x, \tilde{x}) = -\frac{2}{L}\sum_{n=1}^{\infty} \frac{\sin\frac{n\pi\tilde{x}}{L}\sin\frac{n\pi x}{L}}{\left(\frac{n\pi}{L}\right)^2}, \tag{1}$$

while defining the Green's function as a fundamental solution using the delta function, we found that

$$G(x, \tilde{x}) = \begin{cases} -\frac{x}{L}(L-\tilde{x}), & 0 \le x < \tilde{x} \le L \\ -\frac{\tilde{x}}{L}(L-x), & 0 \le \tilde{x} < x \le L. \end{cases} \tag{2}$$

Show that the two functions are equal by expanding (2) in a Fourier sine series. Your result should equal (1).

2. Derive the Green's function for the ODE

$$-\frac{d^2u}{dx^2} + \sigma^2 u = f(x), \qquad u(0) = u(1) = 0.$$

method of characteristics

3. page 533, problem 12.2.3
4. page 533, problem 12.2.5 b, c