Green’s functions

1. In class we derived the Green’s function for the 1D Poisson problem,

\[ \frac{d^2u}{dx^2} = f(x), u(0) = 0 \quad u(L) = 0. \]

Using the method of eigenfunction expansion, we found that

\[ G(x, \tilde{x}) = -2 \frac{L}{\pi} \sum_{n=1}^{\infty} \frac{\sin \frac{n\pi x}{L} \sin \frac{n\pi \tilde{x}}{L}}{(\frac{n\pi}{L})^2}, \quad (1) \]

while defining the Green’s function as a fundamental solution using the delta function, we found that

\[ G(x, \tilde{x}) = \begin{cases} -\frac{x}{L}(L - \tilde{x}), & 0 \leq x < \tilde{x} \leq L \\ -\frac{\tilde{x}}{L}(L - x), & 0 \leq x < \tilde{x} \leq L. & (2) \end{cases} \]

Show that the two functions are equal by expanding (2) in a Fourier sine series. Your result should equal (1).

2. Derive the Green’s function for the ODE

\[ -\frac{d^2u}{dx^2} + \sigma^2 u = f(x), \quad u(0) = u(1) = 0. \]

method of characteristics

3. page 533, problem 12.2.3

4. page 533, problem 12.2.5 b, c