Phys 512 — Prof. P. Berman
Quantum Mechanics II — Winter, 2004

Review Questions

These are review questions which can be used as a study guide. They are **not** *questions which will necessarily appear on the exam.*

Final Exam: Wednesday, April 28 1:30-6:30, Room 335 West Hall

1. How does the rotation matrix differ from the rotation operator?
2. What is a Casimir operator and why is it useful?
3. How does one determine the generators of a group?
4. The generators of both SU(2) and $O^{+}(3)$ have the same Lie algebra. How is this possible if the groups are different.
5. What is an irreducible representation of a group?
6. A hydrogen atom is in the state $\psi_{210}(\mathbf{r})\chi_{\uparrow}$. Calculate the change in this state under an arbitrary rotation.
7. Repeat the calculation of problem 6, but assume the state of the electron is $|n = 2, \ell = 1, j = 1/2\rangle$.
8. A spin 1/2 charged particle is in a magnetic field $\mathbf{B} = B_0\hat{\mathbf{x}} + B\hat{\mathbf{z}}\cos\omega t$. Write down the equations that govern the time development of the state vector. How could this problem be simplified using the rotation operator?
9. Using an infinitesmal transformation determine prove that the components of a spin 1 particle transform as the transpose of the rotation matrix, provided that the components satisfy the normal commutation relations.
10. What is an irreducible tensor operator and how does it transform under rotation?
11. What is the Wigner-Eckart theorem and why is it useful?
12. Find approximate eigenvalues and eigenenergies of the matrix $\begin{pmatrix} 1 & .1 & .2 & .1 \\ .1 & 3 & .3 & .2 \\ .2 & .3 & 5 & .1 \\ .1 & .2 & .1 & 7 \end{pmatrix}$.
13. A one-dimensional oscillator is subjected to a perturbation αx, where α is a constant. Find the lowest order nonvanishing corrections to the eigenergies anth the eigenfunctions. Calculate $\langle x\rangle$ for the new wavefunctions. Will this value change when higher order corrections to the wave functions are included?
14. Show that under a translation $\mathbf{r} \rightarrow \mathbf{r} + \mathbf{a}$ that the position operator transforms as $\hat{\mathbf{r}} \rightarrow \hat{\mathbf{r}} - \mathbf{a}$.
15. Two identical particles are in the states $\psi(1) = .8\psi_1 + .6\psi_2$ and $\psi(2) = .6\psi_1 + .8\psi_2$. Find the density matrix for this two-particle system.
16. What are some of the consequences of the fact that the hydrogen atom has $O^{+}(4)$ symmetry?
17. Evaluate $\langle j \| J \| j\rangle$.
18. Find $\langle \boldsymbol{\mu} \rangle$ for a spin charged 1/2 particle in a magnetic field $\mathbf{B} = B_0\hat{\mathbf{z}} + B\hat{\mathbf{x}}\cos\omega t$ starting in the spin down state at $t = 0$.

19. What is meant by "good" quantum numbers? Give examples related to the hydrogen atom in an external magnetic field.

20. Calculate the fine structure splitting in hydrogen, in terms of matrix elements of the operator $V(r)\mathbf{L}\cdot\mathbf{S}$.

21. Calculate the Zeeman splitting in hydrogen.

22. Find the ground state energy and wavefunctions for three non-interacting particles moving in a one-dimensional harmonic oscillator potential asssuming the particles are (a) spinless bosons, (b) spin 1.2 particles.

23. A hydrogen atom is in its 2P, m=0 subspace. An electric field along the z axis is turned on. Calculate the subsequent wave function assuming (a) the field is turned on suddenly and (b) the field is turned on adiabatically. Assume that the 2S and 2P levels are split by the Lamb shift which is on the order of a GHz.

24. Under what general conditions does Fermi's golden rule hold?

25. An auto-ionizing state is an electronic state in an atom or molecule that is coupled to a continuum by a potential $V(\mathbf{r})$. Calculate the autoioninization rate in terms of matrix elements of $V(\mathbf{r})$.

26. What is the difference betweem ℓs and jj coupling?

27. Classify the ground states of oxygen.

28. Is there a linear Stark effect for the degeneate states of the 3-dimensional isotropic harmonic oscillator. Explain. What type of interaction would couple degenerate states having different angular momentum?

29. How is the Wigner-Eckart theorem used to calculate matris elements of $\mathbf{r}\cdot\mathbf{E}$, where $\mathbf{E}$ is a vector field (not an operator)?

30. What are "large" and "small" components in the Dirac equation?

31. How does electron spin emerge naturally from the Dirac equation? How can one get the Pauli approximation as a limiting form of the Dirac equation?

32. Calculate the free particle eigenfunctions for a Dirac particle.