Phys 512 — Prof. P. Berman
Quantum Mechanics II — Winter, 2004

Problem Set 7
Due Friday, March 5

1. Assume that after diagonalizing a degenerate sub-block the matrix you end up with is

$$\begin{pmatrix} 1-a & 0 & a \\ 0 & 1+a & a \\ a & a & b \end{pmatrix}$$

Show that the corrections to the energies $(1-a, 1+a, b)$ are of order $a^2/|b-a|$ and the corrections to the eigenkets are of order $a/|b-a|$. Choose $a = .1$ and $b = 100$. Calculate the eigenvalues and eigenenergies using nondegenerate perturbation theory and show that the corrections differ from the exact corrections by a significant amount. Why is this so? The moral here is that for large $|b-a|$ you should be content to neglect contributions from outside the degenerate sub-blocks or you must exactly diagonalize a larger matrix.

2-4. Exactly diagonalize the states having $n = 2$ in hydrogen, taking into account the relativistic mass correction, the spin-orbit interaction, and the interaction with a magnetic field. Plot the energy levels as a function of $\beta_0 B/(3W/2)$, where $3W/2 = \frac{1}{32}\mu c^2 \alpha^4$ is the spin-orbit splitting in zero field. For what value of the field are the two contributions equal. **Problem 88-90, parts iv and v**. Note that in part v, the degeneracy is not completely removed by the magnetic field.

5. Problem 99.

6-7. The hyperfine interaction for a hydrogen atom in a given state of n, ℓ, j is given by

$$H_{hf} = \left|E_0^{(0)}\right| \frac{\mu}{M_p} \frac{\alpha^2 g_p}{n^5(2\ell+1)j(j+1)} \frac{\mathbf{J}\cdot\mathbf{I}}{\hbar^2}$$

where M_p is the proton mass, $\mathbf{I}$ is the proton spin, and $g_p = 5.6$ is the proton g factor. In the absence of any external magnetic field, what are the constants of the motion, assigning the total angular momentum $\mathbf{F} = \mathbf{I} + \mathbf{J}$. Obtain the hyperfine structure splitting of the $1S_{1/2}$ $2S_{1/2}$ $2P_{1/2}$ and $2P_{3/2}$ states in MHz. At what values will an external magnetic field produce splittings equal to the hyperfine splitting? For magnetic field interactions much greater than the hyperfine interaction but much less than the fine structure interaction, obtain the energy levels of the $1S_{1/2}$ state (there are 4 levels here) to first order in the hyperfine interaction, considering the hyperfine interaction as a small perturbation compared to the magnetic field interaction [hint: in considering the magnetic field interaction you can neglect the contribution from the nuclear spin - why?]. In the limit that the magnetic field is much less than the hyperfine interaction show that the energy levels are given by $\Delta E = \beta_0 B m_F g_F$ where

$$g_F = g_J \frac{F(F+1) - I(I+1) + J(J+1)}{2F(F+1)},$$

again neglecting the contribution from the nuclear spin in the magnetic field interaction. Without solving the problem exactly, sketch qualitatively, the $1S_{1/2}$ energy levels as a function of magnetic field.