Phys 512 Prof. P. Berman
Quantum Mechanics II Winter, 2004

Problem Set 5

Due Friday, February 13

Midterm Exam: Wednesday, February 18. Room 340 West Hall 5:30-8:30

1-2. Problems T6-T7

3. Prove that $\sum_m (-1)^{j-m} \begin{bmatrix} j & j & k \\ m & -m & q \end{bmatrix} = \sqrt{2j+1}\,\delta_{k0}\delta_{Q0}$. Use this fact to prove that a spherically symmetric Hamiltonian H_0, expanded in terms of the U_k^q given in the previous problem contains only terms proportional to U_0^0.

4. For a state having $J = 1$, express the irreducible components of the density matrix operator, $\rho_k^q(j = 1, j = 1)$ in terms of the $\rho_{1m,1m'}$.

5. Write the operator $\mathbf{r} \cdot \mathbf{E}$ in an irreducible tensor basis, where $\mathbf{r}$ is the position operator and $\mathbf{E}$ is an arbitrary vector (but not an operator).

6. Prove that if $T_{k_1}^{q_1}$ and $T_{k_2}^{q_2}$ are irreducible tensor operators, then the $T_k^q = \sum_{q_1 q_2} \begin{bmatrix} k_1 & k_2 & k \\ q_1 & q_2 & q \end{bmatrix} T_{k_1}^{q_1} T_{k_2}^{q_2}$ form an irreducible tensor operator of rank k. This is especially useful in interactions between two particles, when the individual angular momenta of the particles can be coupled.

7. Problem T9. You can neglect the part of the problem asking you to estimate the hyperfine interaction in terms of the fine structure splitting.