Phys 512 Prof. P. Berman
Quantum Mechanics II Winter, 2004

Problem Set 4
Due Friday, February 6
Midterm Exam: Wednesday, February 18. Room 340 West Hall 5:30-8:30

1. Some transformations under time reversal are $\Theta\mathbf{r}\Theta^{-1} = \mathbf{r}$, $\Theta\mathbf{p}\Theta^{-1} = -\mathbf{p}$, $\Theta\mathbf{L}\Theta^{-1} = -\mathbf{L}$. Prove the first two of these relationship for a particle having spin 0 by taking $\Theta = \Theta^{-1} = K$, where K indicates complex conjugation, and considering the quantities $\Theta\mathbf{r}\Theta^{-1}\psi$ and $\Theta\mathbf{p}\Theta^{-1}\psi$, where $\mathbf{r}$ and $\mathbf{p}$ are operators. Prove that $\Theta^{-1} = \Theta^{\dagger}$ (antiunitary) when $\Theta = K$, even though Θ is an antilinear operator. Also prove that the commutation relation $[x, p_x]$ changes sign under this antilinear operation.

2. Consider the linear group in two dimensions given by the transformations

$$x' = ax + by$$
$$y' = cx + dy.$$

This is a four parameter group assuming a, b, c, d are real. By carrying out an infinitesmal transformation about the identity, obtain the four generators of the group and the Lie algebra associated with these generators. Show that the sum of the square of each of the generators is *not* a Casimir operator of the group (for a *compact* Lie group the sum of the squares of the generators is one of the Casimir operators).

3. Consider a vector $\mathbf{r} = r\hat{\mathbf{j}}$. Show that a passive rotation $R^p(-\gamma, -\beta, -\alpha)$ produces the same relative position of the vector to the new axes as the active rotation $R^A(\alpha, \beta, \gamma)$ does relative to the original axes. You need do this only for $(\alpha, \beta, \gamma) = (\pi/2, \pi/2, 0)$

4. Prove that the Clebsch-Gordon coefficient $\begin{bmatrix} j_1 & j_2 & j \\ m_1 & m_2 & m \end{bmatrix}$ vanishes unless $m = m_1 + m_2$ and $|j_1 - j_2| \le j \le j_1 + j_2$.

5. Problem T1. You can use tables or the programs in Mathematica.

6. Problem T8. There are 2 problems T8 in the handout. This is the second T8.

7. Prove the orthogonality relations for the Clebsch-Gordon coefficients

$$\sum_{m_1, m_2} \begin{bmatrix} j_1 & j_2 & j \\ m_1 & m_2 & m \end{bmatrix} \begin{bmatrix} j_1 & j_2 & j' \\ m_1 & m_2 & m' \end{bmatrix} = \delta_{mm'}\delta_{jj'};$$

$$\sum_{j, m} \begin{bmatrix} j_1 & j_2 & j \\ m_1 & m_2 & m \end{bmatrix} \begin{bmatrix} j_1 & j_2 & j \\ m_1' & m_2' & m \end{bmatrix} = \delta_{m_1 m_1'}\delta_{m_2 m_2'}$$

and obtain the corresponding equations for the 3-J symbols.