Phys 512 Prof. P. Berman
Quantum Mechanics II Winter, 2004

Problem Set 2

Due Friday, January 23

1. Problem 86d from supplemental handout.

2. For a two spin system, prove that $Tr(\rho^2) \leq 1$. To prove this, it may be helpful to prove that $|\rho_{\alpha\beta}|^2 \leq \rho_{\alpha\alpha}\rho_{\beta\beta}$. In these expressions α and β can be spin up or spin down and ρ is the density matrix defined by $\rho = \sum_j \rho^{(j)}$ where $\rho^{(j)}$ is an indivual spin density matrix. This result can be extended to an arbitrary N particle system with an arbitrary number of states α and β, but you are not asked to prove this.

3-4. Consider an ensemble of spin 1/2 particles in a constant magnetic field along the z-axis. At $t = 0$ all the particles are in the spin down state. A short pulse of magnetic field radiation is applied at $t = 0$ along the x-direction. Assume the carrier frequency of the pulse is sufficiently close to resonance that the pulse can be considered to be in resonance with the transition. Also, for simplicity, take the pulse envelope to be square with $\omega_x T = \pi/2$, where T is the pulse duration. Use the Bloch vector picture to calculate u, v, and w for times $t > 0$ (You can assume that T is sufficiently short so you can set $t = 0$ immediately after the pulse. Also calculate $\langle \vec{\sigma} \rangle$ and show that the transverse components of $\vec{\sigma}$ oscillate (don't forget that .u, v, and w are defined in the rotating frame, whereas $\langle \vec{\sigma} \rangle$ is defined for the total wave function). This is the principle of nuclear magnetic resonance.

Now imagine that, owing to impurities or stray fields, there is a Gaussian *distribution* of B_0 along the z axis. The pulse is sufficiently short to excite all the spins equally, but, following excitation there is a distribution of detunings

$$W(\delta) = \sqrt{\frac{1}{\pi\Delta^2}}\, e^{-(\delta/\Delta)^2}.$$

Calculate density matrix elements for the ensemble of spins for times $t > 0$ and show that the off diagonal density matrix elements decay away. The underlying process here is known as *inhomogeneous broadening*.

5. Problem 86a.

6. Problem 86b.