Phys 512 Prof. P. Berman
Quantum Mechanics II Winter, 2004

Problem Set 12
Due Friday, April 9
This is the last problem set for the semester. The last recitation will be on April 8, although there will be a review session on Tuesday, April 27, from 4-5:30 in 4246 Randall.

Final Exam: Wednesday, April 28 1:30-6:30 Room 335 West Hall

1. The first excited state of Helium has the configuration $1s^1 2s^1$ or $1s^1 2p^1$. In a rough approximation assume the wave functions are given by hydrogenic wave functions by neglecting the $e^2/|\mathbf{r}_1 - \mathbf{r}_2|$ term in the Hamiltonian. Show that the first excited state in this approximation is 16-fold degenerate and find the energy E_2 of this state. Now consider the $e^2/|\mathbf{r}_1 - \mathbf{r}_2|$ as a perturbation. Using properly symmetrized wave functions, show that the some of the degeneracy is broken and that the energy of the ^3S and ^1S states are given by

$$E(^1S, ^3S) = E_2 + (E_d(s) \pm E_e(s))$$

where the *direct* and *exchange* integrals are given by

$$E_d(s) = \int \psi^*_{100}(\mathbf{r}_1)\psi^*_{200}(\mathbf{r}_2)\frac{e^2}{|\mathbf{r}_1 - \mathbf{r}_2|}\psi_{100}(\mathbf{r}_1)\psi_{200}(\mathbf{r}_2)d\mathbf{r}_1 d\mathbf{r}_2$$
$$E_e(s) = \int \psi^*_{100}(\mathbf{r}_1)\psi^*_{200}(\mathbf{r}_2)\frac{e^2}{|\mathbf{r}_1 - \mathbf{r}_2|}\psi_{100}(\mathbf{r}_2)\psi_{200}(\mathbf{r}_1)d\mathbf{r}_1 d\mathbf{r}_2$$

while the energy of the ^3P and ^1P states are given by

$$E(^1P, ^3P) = E_2 + (E_d(p) \pm E_e(p))$$

where

$$E_d(p) = \int \psi^*_{100}(\mathbf{r}_1)\psi^*_{210}(\mathbf{r}_2)\frac{e^2}{|\mathbf{r}_1 - \mathbf{r}_2|}\psi_{100}(\mathbf{r}_1)\psi_{210}(\mathbf{r}_2)d\mathbf{r}_1 d\mathbf{r}_2$$
$$E_e(p) = \int \psi^*_{100}(\mathbf{r}_1)\psi^*_{210}(\mathbf{r}_2)\frac{e^2}{|\mathbf{r}_1 - \mathbf{r}_2|}\psi_{100}(\mathbf{r}_2)\psi_{210}(\mathbf{r}_1)d\mathbf{r}_1 d\mathbf{r}_2.$$

You need not evaluate these integrals (but you can if you wish). Their values are $E_d(s) = 11.4$ev, $E_e(s) = 1.2$ev, $E_d(p) = 13.4$ev, $E_e(s) = 0.94$ev. Draw an energy level diagram for the first excited states using these values and your value of E_2.

2. Problem T11.

3. Problem T13.

4. Problem T14. Estimate the temperatures at which rotational and vibrational degrees of freedom are ”frozen out”.

5. Prove explicitly that if two states having $\ell = 1$ and the same n are coupled, the resulting states having angular momentum $L = 0, 1, 2$ are either symmetric or antisymmetric on particle exchange. Problem 113.

6. Prove explicitly that the state $\psi(1,2,3) = \psi_{11}(1)\psi_{11}(2)\psi_{10}(3) \uparrow (1) \downarrow (2) \uparrow (3)$ when properly antisymmetrized is an eigenstate of L^2 with eigenvalue $6\hbar^2$ and an eigenstate of S^2 with an eigenvalue $(3/4)\hbar^2$. The $\psi_{\ell m}$ are the spatial wave functions. Thus this state is a ^2D state as claimed in class.