Phys 512 Prof. P. Berman
Quantum Mechanics II Winter, 2004

Problem Set 11
Due Friday, April 2
Final Exam: Wednesday, April 28 1:30-6:30 Room 335 West Hall

1. Consider two, identical, noninteracting spin 1/2 particles moving in a 1-D infinite square well potential. Find the ground state energy and ground state wave function. Prove that the first excited state is 4-fold degenerate. Find the energy of these 4 states and the wave functions.

2. For the same potential as in problem 1, calculate the ground state of three, identical spin 1/2 particles. Can this state be written as a product of a symmetric spatial state and an antisymmetric spin state? Explain.

3. Prove that the permutation operator P_{ij} is Hermitian.

4. Imagine that you have two identical bosons separated by a distance that is much larger than their de Broglie wavelength (one in Ann Arbor and one in Chicago, for example). An appropriate symmetrized wave function is $\psi(\mathbf{r}_1,\mathbf{r}_2) = [\psi_a(\mathbf{r}_1)\psi_c(\mathbf{r}_2) + \psi_a(\mathbf{r}_2)\psi_c(\mathbf{r}_1)]/\sqrt{2}$ or a simple product $\psi = \psi_a(\mathbf{r}_1)\psi_c(\mathbf{r}_2)$, where ψ_a is the wave function in Ann Arbor and ψ_c is the wave function in Chicago.Show that if one asks for the probabiltiy of measuring either of these bosons in Ann Arbor with *either* $\mathbf{r}_1 = \mathbf{r}_a$ or $\mathbf{r}_2 = \mathbf{r}_a$, it makes no difference whether we use the symmetrized wave function or a factorized wave function of the form $\psi(\mathbf{r}_a,\mathbf{r}_c) = \psi_a(\mathbf{r}_a)\psi_c(\mathbf{r}_c)$, where $\mathbf{r}_c$ is the coordinate of the boson in Chicago. In other words, as long as the wave functions of identical particles *never* overlap, it is not necessary to symmetrize the wave function - the particles can be considered to be distinguishable.

5. As shown in class, the interaction of a resonant field with two identical atoms, each having ground state 1 and excited state 2 separated by frequency ω_0 leads to equations for the state amplitudes (in an interaction representation)

$$\dot{a}_{11} = -i\chi(a_{12} + a_{21})$$
$$\dot{a}_{22} = -i\chi(a_{12} + a_{21})$$
$$\dot{a}_{12} = -i\chi(a_{11} + a_{22})$$
$$\dot{a}_{21} = -i\chi(a_{11} + a_{22})$$

where χ is a coupling constant and the total wave function is

$$|\psi(t)\rangle = a_{11}|11\rangle + (a_{12}|12\rangle + a_{21}|21\rangle)e^{-i\omega_0 t} + a_{22}|22\rangle e^{-2i\omega_0 t}.$$

Show that only symmetric states are produced if, at $t = 0$, the system is in state $|11\rangle$. Since one of these symmetric states is an entangled state it might seem like the classical field has produced entanglement. Show this is not the case by solving the equations and proving that $|\psi(t)\rangle$ can be written as a product state for the 2 particles.

6. The contribution to the decay rate of spontaneous emission from radiation having $\hat{\theta}$ or $\hat{\phi}$ polarization is proportional to

$$I_\lambda(\mathbf{k}) = \left| \sum_{q,m_f} [\epsilon_\lambda(\mathbf{k})]_q \begin{bmatrix} J_f & 1 & J_e \\ m_f & q & m_e \end{bmatrix} \right|^2$$

where

$$\epsilon_1(\mathbf{k}) = \hat{\theta} = \cos\theta\cos\phi\hat{x} + \cos\theta\sin\phi\hat{y} - \sin\theta\hat{z}$$
$$\epsilon_2(\mathbf{k}) = \hat{\phi} = -\sin\phi\hat{x} + \cos\phi\hat{y}$$

and the large square bracket is a Clebsch-Gordan coefficient. For $J_f = 0, J_e = 1$, and $m_e = 0$, show that only the $\hat{\theta}$ component contributes and this is consistent with a dipole pattern. For $J_f = 0, J_e = 1$, and $m_e = 1$, show that both the $\hat{\theta}$ and $\hat{\phi}$ components contribute and that the results are consistent with circular polarization only for $\theta = 0, \pi$. Finally for For $J_f = 1$, $J_e = 2$, and $m_e = 0$, show that both the $\hat{\theta}$ and $\hat{\phi}$ components contribute and the pattern is consistent with a dipole pattern plus an isotropic background.