Phys 512 — Prof. P. Berman
Quantum Mechanics II — Winter, 2004

Problem Set 10
Due Friday, March 26
Final Exam: Wednesday April 28, 1:30-6:30 Room 335 West Hall

1. In Fermi's Golden Rule, one encounters an integral of the form

$$I = \int_{-\epsilon/2}^{\epsilon/2} dE \sin^2(Et/2\hbar)/E^2.$$

Show that this integral is proportional to

$$A = t\int_{-x}^{x} dz \sin^2(z)/z^2,$$

where $x = \epsilon t/4\hbar$. Plot A/t as a function of x. How is your result related to the validity condition for Fermi' Golden Rule? For what values of x does Fermi's Golden Rule hold? For $x \leq 1$, how does the transition probability vary with t?

2. Consider a two-level atom interacting with a classical radiation field

$$\mathbf{E} = E_0 \boldsymbol{\epsilon} \cos(\omega t)$$

in the resonance approximation, $|\delta| = |\omega_0 - \omega| \ll \omega_0 + \omega$. Use perturbation theory to find the upper state probability amplitude a_2 as a function of time, but include a term $-(\Gamma/2)a_2$ in the equation for $\dot{a}_2$ to account for spontaneous emission. Assume the atom is in the lower state at $t = 0$. Show that the upper state population reaches a steady state value and plot this as a function of δ. How is the absorption of the field related to this upper state population?

3. Problem 120. Take an interaction potential $-\wp \cdot \mathbf{E}$, where $\wp$ is the atomic dipole moment operator and $\mathbf{E}$ is an applied electric field.

4-5. Problem 116. Assume that the impact parameter is much larger than the Bohr radius, but that $b/v \ll \hbar/|E_0|$, where $-|E_0|$ is the ground state energy of hydrogen.