Introductory Quantum Mechanics: A Traditional Approach Emphasizing Connections with Classical Physics

Supplementary Problems

Supplementary problems will be added from time to time.

Chapter 19

Consider a one-dimensional potential for which a wave packet can be sent in from the right or left. Suppose that the (amplitude) reflection coefficent for a wave incident from the left is R and the transmission coefficient is T. For a wave incident from the right, the (amplitude) reflection coefficent for a wave incident from the left is R' and the transmission coefficient is T'. If the Hamiltonian is time reversal invariant, prove the Stokes relations,

$$T' = T; \quad R' = -R^*T/T^*$$

and show that these relations are consistent with $R' = -R$ for the step potential (with energy above the step potential) and $R' = R$ for the square well potential. [Hint: Consider the time reversed scattering of the case when the particle is incident from the left to prove that

$$\begin{aligned} T^*T' + R^*R &= 1; \\ T^*R' + R^*T' &= 0.] \end{aligned}$$

Chapter 13

Consider the $J = 0 - 1 - 0$ cascade emission of a single atom with radiation emitted along the y axis. Assume that the single photon state emitted on the $0-1$ transition propagates in the $+y$ direction and that on the $1-0$ transition in the $-y$ direction. The polarization of the emitted radiation is in the entangled state

$$|\psi\rangle = \frac{|x_1x_2\rangle + |z_1z_2\rangle}{\sqrt{2}},$$

where x and z indicate the polarization direction of the emitted radiation. Now suppose that there are polarizers placed in the path of each beam whose axes make an angle α_j with the z axis $(j = 1, 2)$. Define new sets of basis vectors by

$$\begin{aligned} |H_j\rangle &= \cos\alpha_j\,|z_j\rangle + \sin\alpha_j\,|x_j\rangle\,; \\ |V_j\rangle &= -\sin\alpha_j\,|z_j\rangle + \cos\alpha_j\,|x_j\rangle\,. \end{aligned}$$

Write the state vector $|\psi\rangle$ in terms of these new basis kets and show that when the detectors are set at the same angles, there is perfect correlation of any observed signals; that is if a signal is detected behind detector 1 in a given experiment, there will be a signal detected behind detector 2 in that experiment.

Also show that if the radiation is emitted sometimes into$|x_1 x_2\rangle$ and sometimes into $|z_1 z_2\rangle$ there would no longer be perfect correlation if the detectors are set equal to the same *arbitrary* angle α.

Chapter 21

1. Prove that Eq. (21.62) is correct for

$$\left\langle n=2,\ell=0,s=1/2,j=1/2,m_j'\right| \hat{z}\left|n=2,\ell=1,s=1/2,j=1/2,m_j\right\rangle,$$

where the ket labels are those of hydrogen. In other words evaluate both sides of Eq. (21.62) with $\hat{A}_1^0=\hat{z}$ and show that they are equal.

2. Prove that both $\hat{F}_z$ and $\hat{F}^2$ commute with the hyperfine interaction potential

$$\hat{H}_{hf}=\frac{\mu_0 e^2 g_p}{8\pi m_e m_p}\left[\frac{8\pi\delta(\mathbf{r})}{3}\mathbf{S}\cdot\mathbf{I}+\frac{3\left(\mathbf{u}_r\cdot\mathbf{S}\right)\ \left(\mathbf{u}_r\cdot\mathbf{I}\right)-\mathbf{S}\cdot\mathbf{I}}{r^3}+\frac{\mathbf{L}\cdot\mathbf{I}}{r^3}\right].$$

Hint: The orbital angular momentum operator does not commute with $\mathbf{u}_r$, considered as an operator.