General Relativity

1. In Newtonian physics, \( m_i = m_g \) was an "accident" or "coincidence." The fact that all objects have the same acceleration near the surface of the earth led to the conclusion that inertial mass equals gravitational mass.
2. In a local free-falling frame, gravity is absent.
3. In "outer space" one can simulate a uniform gravitational field by going into an accelerating frame.
4. The above ideas led Einstein to the General Theory of Relativity.
   a. Principle of General Covariance — laws of physics take the same form in all reference frames.
   b. Principle of Equivalence: This can be stated three different ways:
      (i) \( m_i = m_g \)
      or
      (ii) Locally, one can eliminate effects of gravity by going into a free falling frame
      or
      (iii) Locally, one can create the effect of a uniform gravitational field by going into an accelerating frame.
      Note that if \( m_i \neq m_g \), effects of gravity and acceleration would not be equivalent. Why?
5. Using the hypothesis that any effects seen in an accelerating reference frame will also be seen in a true gravitational field, one can show that:
   a. light is deflected in a gravitational field
   b. clocks run slower in a stronger gravitational field
   c. clock synchronization and rigid rods are impossible in a gravitational field.
6. Since no rigid rods and synchronized clocks exist in a gravitational field, one must decide how to make measurements.
   a) In a "local" region, assume rods rigid (if at rest relative to each other) and that the speed of light is constant (use these facts to synchronize clocks).
   b) By using the clocks and meter sticks to measure events over large or "global" distance, you can measure all the effects discussed in "5" above.
7. An alternative interpretation is that light is always moving on "straight lines" and that we chose the wrong way to measure. In this view, space-time is curved and light and matter just move on their natural paths (geodesics) in curved space time.
8. The curvature of space-time is expressed by the metric tensor \( g_{\mu\nu} \). In flat (Euclidean) space \( g_{11} = g_{22} = g_{33} = 1; g_{44} = -1 \) and all others are zero. If there is no reference frame in which the metric tensor takes on these
values, then some mass must exist that is producing a true gravitational field or, alternatively, curving space-time. In other words, it is impossible to find a reference frame in which a true gravitational field can be *globally* transformed away — this can only be done in small regions of space-time.

9. Einstein field equations show how the mass and energy distribution determines $g_{\mu\nu}$ and how to calculate trajectories of particles and light in curved space-time. Gravity is eliminated as a force — particles move on geodesics in curved space-time.

10. A measure of the curvature of space time comes from the Schwarzschild solution of the Einstein field equations. For a sphere of mass $M$, the Schwarzschild radius is defined as the radius the sphere would have so that the escape velocity would equal the speed of light at its surface. One finds

$$R_s = \frac{2GM}{c^2}$$

For the Earth, $R_s(\text{Earth}) \approx 1 \text{ cm}$
For the Sun, $R_s(\text{Sun}) \approx 1 \text{ km}$

A measure of the importance of the curvature of space time [that is, a measure of the *correction* from Newton's laws] is the parameter

$$\alpha = \frac{R_s}{R}$$

where $R$ is the distance from the sphere which is assumed to have radius $a$ ($R \geq a$).

Near the surface of the sun, $R_s/R \approx 10^{-6}$. This means that Newton’s laws are wrong by one part in a million near the sun’s surface. At Mercury’s orbit, $R_s(\text{sun})/R(\text{radius of Mercury’s orbit}) \approx 10^{-8}$. This gives a small contribution to the precession of Mercury’s orbit ($\approx 40 \text{ sec of arc per century}$). Thus the deviations of Newton’s laws from General Relativity occur only for large masses contained in small volumes, that is, only if $\alpha$ becomes comparable with unity.

11. We make measurements as in “6” above. Some experimental tests of General Relativity are:

a. Bending of starlight as it passes by sun. (1.7“ of arc)
b. Gravitational red shift of atoms emitting near the surface of the sun (shift of 1 part in a million).
c. Precession of perihelion of Mercury’s orbit (43“ of arc per century).
d. Slowing down of binary pulsars (owing to emission of gravitational waves).
e. Gravitational lensing.

The first three are the early tests of the predictions of the theory.