Phys 511 Prof. P. Berman
Quantum Mechanics I Fall, 2003

Problem Set 8
Due Friday, October 31

1. Consider a classical free particle moving in the z direction at some distance b from the z axis. Show that the energy and the angular momentum of the particle fix the value of the impact parameter b. Classically, what range of r and θ are allowed. Now use a 3D plot to graph the probability distribution of a quantum particle as a function of kr and θ for $\ell = 20$ [you should plot $|\psi_{\ell,m=0}(kr)|^2(kr)^2\sin(\theta)$]. Carefully compare the quantum result with the classical one.

2. Prove that under an inversion of coordinates, $\mathbf{r} \to -\mathbf{r}$, $Y_\ell^m(\theta,\phi) \to (-1)^\ell Y_\ell^m(\theta,\phi)$. Thus the $Y_\ell^m(\theta,\phi)$'s are eigenfunctions of the parity operator with eigenvalues $(-1)^\ell$.

3-4. For an attractive square well potential, $V(r) = -V_0 < 0, r < a$; $V(r) = 0, r > a$, obtain the equations that can be used to solve for the bound state energy levels. Obtain explicit transcendental equations for $\ell = 0$ and $\ell = 1$ and find the minimum value of $\beta^2 = 2mV_0/\hbar^2$ that supports a bound state for these values of ℓ. Why does the minimum depth needed to support a bound state increase with increasing ℓ? The proton and neutron in a deuteron can be modeled as interacting with a square well potential with $V_0 = 37$ Mev and $a = 2.0 \times 10^{-15}$ m. Prove that only one bound state exists and calculate its energy, which is the binding energy of the deuteron (the actual value, which differs somewhat from this calculation, is 2.22 Mev).

5. Plot $\rho^2|R_{n,l}(\rho)|^2$ for the $n = 0$, $n = 1$, and $n = 2, \ell = 0, 2$ eigenstates of the 3D SHO (ρ is the same as the dimensionless length ξ used in class). Interpret your results in terms of the effective potential. Calculate $\langle\rho\rangle$ and $\langle\rho^2\rangle$ in an eigenstate of the oscillator. For fixed n, how do these quantities vary with ℓ?