Phys 511 Prof. P. Berman
Quantum Mechanics I Fall, 2003

Problem Set 7

Due Friday, October 24

Midterm Exam: Monday October 20, 5:30-7:00 in Room 340 West Hall

All material in Chaps. 1-6 in Merzbacher

1. Problem 36-37.

2. Problem 40. There is a misprint in this problem. You are asked to prove that $\frac{\hbar}{i}\frac{\partial}{\partial r}$ is not Hermitian but that $\frac{1}{2}[(\frac{\mathbf{r}}{r})\cdot\mathbf{p}+\mathbf{p}\cdot(\frac{\mathbf{r}}{r})]$ is Hermitian.

3. Consider the potential in 2-D with $V(\rho)=0$ for $\rho<a$ and $V(\rho)=\infty$ for $\rho>a$. An electron moves in this potential. Find the three lowest eigenenergies (in eV) and eigenfunctions for $a=10^{-8}$ cm.

4-5. (Optional, but that doesn't mean you shouldn't try it). Consider a free particle moving inside a conical "box" with $V=0$ for $\theta<\beta$ and $r<a$, and $V=\infty$ elsewhere. As discussed in class, the angular part of the wave function for can be written as an associated Legendre function $P_\nu^m(x=\cos\theta)e^{im\phi}$, where m is an integer, but ν is no longer restricted to be an integer if $x=-1$ is not included in the space. Find the three lowest energy eigenvalues ka for this system when $\beta=\pi/4$ ($k^2=2mE/\hbar^2$). [Hint: Mathematica evaluates the LegendreP[n,m,x] and BesselJ[n+1/2,ka] functions for arbitrary arguments. You must first find the lowest values of ν consistent with the boundary conditions on θ. Then show that solutions of the radial equation vary as $J_{\nu+1/2}(kr)$ and find values of ka that satisfy the boundary condition at $r=a$. I found the lowest energy states correspond to the lowest root of $P_\nu^0(\cos\beta)=0$, the second lowest energy to the lowest root of $P_\nu^1(\cos\beta)=0$, the third lowest energy to the lowest root of $P_\nu^0(\cos\beta)=0$, the fourth lowest energy to the lowest root of $P_\nu^2(\cos\beta)=0$, and the fifth lowest energy to the second lowest root of $P_\nu^0(\cos\beta)=0$. Recall for each υ, there are an infinite number of solutions of $J_{\nu+1/2}(ka)=0$. Why?] Is there any energy degeneracy in this problem?