Phys 511 Prof. P. Berman
Quantum Mechanics I Fall, 2000

Problem Set 6
Due Friday, October 17
Solutions to this problem set will be handed out on Friday, October 17.
Midterm Exam: Monday October 20, 5:30-7:00 in Room 340 West Hall
All material in Chaps. 1-6 in Merzbacher

1. Problem 28 (b). Plot the probability density as a function of time for $\omega t = 0, \pi/2, \pi, 2\pi$. Go to the web site webphysics.davidson.edu/applets/applets.html and then to the topic QM Time. Put in a harmonic oscillator potential and your original wave function to see its time evolution.

2. Problem 29. Hint: What is the expectation value of any time-independent operator in an eigenstate of a Hamiltonian?

3-4. In a harmonic oscillator potential, the initial wave function is

$$\psi(x,0) = Ne^{ip_0x/\hbar}e^{-\alpha^2(x-x_0)^2/2},$$

with $\alpha^2 = m\omega/\hbar$ and N is a normalization constant. This is a minimum uncertainty packet - the ground state wave packet displaced by x_0. Calculate $\psi(x,t)$ and show that the center of the packet follows the classical trajectory and that the packet does not spread in time. This is called a **coherent state** wave packet. Hint: Proceed as in any time-dependent problem. Find the expansion coefficients using $\psi(x,0)$ and then obtain $\psi(x,t) = \sum_n c_n\psi_n(x)e^{-iE_nt/\hbar}$. The integrals and sums that appear can be evaluated using tables of integrals, the generating function, Eq. (5.37) in Merzbacher, or the recursion relations. The final result is

$$\psi(x,t) = N\exp\left\{ \begin{array}{c} -\frac{\alpha^2}{2}[x - Q\cos(\omega t+\delta)]^2 - ixQ\alpha^2\sin(\omega t+\delta) \\ +\frac{\alpha^2Q^2}{4}i[\sin 2(\omega t+\delta) - \sin 2\delta] - i\frac{\omega t}{2}) \end{array} \right\}$$

where

$$Qe^{-i\delta} = x_0 + \frac{ip_0}{\hbar\alpha^2}.$$

At the web site, webphysics.davidson.edu/applets/applets.html (topic QM Time), put in an initial Gaussian packet for a SHO potential to see that the probability density does not spread in time.