Phys 511 — Prof. P. Berman
Quantum Mechanics I — Fall, 2003

Problem Set 5
Due Friday, October 10
The midterm exam is scheduled for Monday, October 20 from 5:30-7:00. Room 340 West Hall.

1. Consider the 1-D Hamiltonian

$$H = \frac{p^2}{2m} + \beta|x|^{\mu}$$

where μ is a real number greater than 0. Estimate the ground state energy. Show that, in the classical limit of large energy, the energy levels vary as n^{α}, where n is an integer. Find α.

2. Consider a step function barrier $V(x) = V_0\Theta(x)$, where $\Theta(x) = 1$ for $x > 0$ and is zero for $x < 0$. The wave packet is chosen so it is centered at $x = -x_0$ at time $t = 0$, has a fairly well defined velocity $\mathbf{v}_0 = \hbar k_0 \hat{\mathbf{i}}/m$, where m is the particle's mass, and does not spread appreciably on the time scale of the problem. Choose a packet having average energy greater than V_0 and show that the transmitted packet is centered at $x = (k'/k_0)(-x_0 + v_0 t)$ for times $t > x_0/v_0$, where $k'^2 = 2m(E - V_0)/\hbar^2$. (Hint: Expand the initial wavefunction in terms of plane wave states as was done in class. To find $\psi(x,t)$ for $x > 0$, the eigenfunction has a factor $e^{ik'x}$. Expand k' about $k = k_0$, keeping only terms to order $(k - k_0)$, and express the result in terms of the wave packet at time $t = 0$.) Show that the result agrees with the classical limit and, in contrast to the case of $E < V_0$, there is no time delay.

Super optional. Revisit the case $E < V_0$ and take an initial wave function that is compact, that is it truly vanishes for $x > 0$ at $t = 0$ (you might take a Gaussian times a step function). Numerically find $\psi(x,t)$ to determine how fast and how the matter wave penetrates the barrier.

3. Problem 16.

4. Prob 21.

5. Prob 24.