Phys 511 Prof. P. Berman
Quantum Mechanics I Fall, 2003

Problem Set 2
Due Friday, September 19

1. Consider a momentum state wave packet

$$\varphi(\mathbf{k}) = |\varphi(\mathbf{k})|e^{i\theta(\mathbf{k})} = \left(\frac{\sigma^2}{2\pi}\right)^{3/4} e^{-|\mathbf{k}-\mathbf{k}_0|^2\sigma^2/2} e^{i\mathbf{k}\cdot\mathbf{r}_0} e^{i\left(\frac{\hbar k^2}{2m}\right)T}.$$

Calculate $|\psi(\mathbf{r},t)|^2$ and show that the center of the wave packet is located at $\mathbf{r} = \mathbf{v}_0 t - \nabla_{k_0}\theta$ and that the minimum width of the wave packet occurs at time $t = \frac{1}{3}(m/\hbar)\nabla^2_{k_0}\theta$. Thus, the phase of the wave function in momentum space determines where the packet is located at $t = 0$ and the time for which the width of the packet is a minimum. Note: No expansion of $\theta(\mathbf{k})$ is needed here since it is already expressed as a series.

2. Problem 12. The mass of the particles for both wave functions is the same. What is the significance of this result?

3. Problem 6. The last sentence should start "Is it" rather than "It is"

4-5. A wave packet is incident on a periodic transmission grating with transmission function $T(x)$ that can be expanded in a Fourier series as

$$T(x) = [T(x)]^2 = \sum_{n=-\infty}^{\infty} \eta_n e^{2\pi i n x/d}.$$

The grating period is d. Neglect the wave packet motion in the y direction and assume the motion in the z direction can be considered to be classical, such that at a time t following the scattering by the grating, the wave packet is located at $z = vt$, where v is the velocity in the z direction. In other words, consider the one dimensional wave packet for which

$$\psi(x,0) = \frac{1}{\sqrt{d}} T(x),$$

normalized such that $\frac{1}{d}\int_0^d [T(x)]^2 dx = 1$. Calculate $\psi(x, t = z/v)$ and show that at integral multiples of the so-called Talbot length

$$z_T = 2d^2/\lambda_{dB}$$

that the wave function is exactly equal to $\psi(x,0)$. Also show that at half integral multiples of the Talbot distance, the initial wave packet is reproduced, but shifted by half a period. The Talbot effect has been observed using matter waves [see Chapman et al., Phys. Rev. A **51**, R14 (1995); Nowak et al., Optics Lett. **22**, 1430 (1997)].

6. Problem 5. Note that this problem is done in Merzbacher, Section 4.7. the next to last sentence should read "and find ϕ explicitly for a free particle."