Phys 511 Prof. P. Berman
Quantum Mechanics I Fall, 2003

Problem Set 11

Due Friday, November 21

1-2. For the hard sphere, $V(r) = 0, r > a$; $\infty, r < a$, calculate the phase shifts. Use a computer program to evaluate the phase shifts as a function of ℓ for $ka = 0.1, 10, 100$. You need only go from $\ell = 0$ to $\ell = ka + 10$ in each case. By using the computer to carry out the sum, obtain the total cross section in each case (in units of a^2) and compare it with π. In the case of $ka = 100$, use the computer to calculate and plot the differential scattering cross section for $\theta < 0.25$ and in the range $\pi/4 < \theta < 3\pi/4$ (both in units of a^2). Interpret your results.

3. For an attractive potential, $V(r) = -V_0, r < a; 0, r > a$, we have shown that the phase shifts are given by

$$\tan\delta_\ell = -\left[\frac{x' j_\ell(x) j_\ell'(x') - x j_\ell(x') j_\ell'(x)}{x n_\ell'(x) j_\ell(x') - x' j_\ell'(x') n_\ell(x)}\right],$$

where $x^2 = 2\mu E a^2/\hbar^2$, $x'^2 = 2\mu(E + V_0)a^2/\hbar^2$, and $\beta^2 = 2\mu V_0 a^2/\hbar^2$. Obtain the differential and total cross section in the limit that $x \ll 1$ (consider only $\ell = 0$). Under what conditions does the cross section $\ell = 0$ contribution to the cross section vanish?

4. For the same potential as problem 1, numerically evaluate the phase shifts for $ka = 10$ and $\beta = 10$. For these values plot the differential cross section (in units of a^2). Note the diffraction peak for $\theta \lesssim 0.2$ evidence for a glory effect near $\theta = \pi$. Super optional: Choose a large value of x (100?, 1000?) and $n = \sqrt{\frac{E+V_0}{E}} = 4/3$. Sum the partial waves to show explicitly the rainbow effect The classical rainbow angle occurs at a deflection angle $\Theta = 4r - 2i$ where $\cos i = \sqrt{\frac{n^2-1}{3}}$ and $n \sin r = \sin i$.

5. For the potential of problem 1 with $\beta = 40$, consider scattering for a fixed partial wave, $\ell = 20$. Show that as a function of x, there is a resonance at $x = 18.3$ and a superfine resonance at $x = 7.84445301224024$. Interpret this result, using the effective potential.