

Problem Set 10

Due Friday, November 14

1-2. Using the relationship

$$\Theta = \pi - 2b \int_a^\infty \frac{dr}{r^2 \sqrt{1 - \frac{b^2}{r^2} - \frac{V(r)}{E}}}$$

show that the classical differential cross section for hard sphere scattering by an infinitely repulsive sphere having radius R is $R^2/4$ and that for a potential $V = \pm\alpha/r$ is

$$\frac{d\sigma}{d\Omega} = \left(\frac{\alpha}{4E \sin^2(\theta/2)} \right)^2$$

3-4. Now consider classical scattering by the potential $V(r) = -V_0, r < a; 0, r > a$, where V_0 is positive. For a particle of mass m having energy $E > 0$, explain why rainbow and glory (maximum in the scattering at $\theta = \pi$) scattering cannot occur. To do this, show that classical cross section is given by

$$\frac{d\sigma}{d\Omega} = \frac{n^2 a^2 (n \cos[\theta/2] - 1)(n - \cos[\theta/2])}{4 \cos[\theta/2] (1 + n^2 - 2n \cos[\theta/2])^2},$$

where $n = \sqrt{1 + (\beta/x)^2}$, $x^2 = 2\mu E a^2 / \hbar^2$, $\beta^2 = 2\mu V_0 a^2 / \hbar^2$, and $\cos[\theta/2] > 1/n$. This is analogous to scattering by a sphere having index of refraction n , but *neglecting* any internal reflections. The quantum or optical problem also can be viewed as scattering by a sphere having index of refraction n , but now reflection and transmission occurs at each surface, even in the geometrical or "classical" limit. As a result, both rainbow and glory scattering can occur in the quantum problem. This is one case where the effective potential for the classical problem does not reveal the richness of the scattering behavior found in the wave world. [Hint: Obtain an equation for the relationship between the deflection angle and the impact parameter using simple geometric optics.]

5. Under what conditions can rainbow and glory scattering occur in classical scattering. Give specific examples.

6. For a potential having a range a , estimate the maximum value of ℓ for which the phase shifts are nonvanishing. For a potential with an *infinite* range that varies as r^{-s} , $s > 2$, estimate the maximum value of ℓ for which the phase shifts are nonvanishing (Hint: use the uncertainty principle to find the largest impact parameter for which a classical picture of the scattering is still possible). For a potential with an *infinite* range that varies as r^{-s} , $s > 2$, estimate the quantum mechanical total cross section. What is the classical total cross section for such a potential?